

PHYSICS 115/242

Homework 3

Due in class, Monday, April 18.

Note: As always, the answers to the questions must include the computer code and output, in addition to any writing that might be needed. If you are not sure what is required please ask me.

1. Sketch the function

$$x^3 - 5x + 3 = 0.$$

Determine the two positive roots to 4 decimal places using the bisection method.

Note: You first need to bracket each of the roots

2. Take the two roots that you found in the previous question (accurate to 4 decimal places) and “polish them up” to 14 decimal places using the Newton-Raphson method.
3. Determine the positive solution of

$$x = \tanh(2x)$$

to 12 decimal places using the secant method.

4. Determine $\sqrt{2}$ to 14 decimal places using the Newton-Raphson method.
5. Consider the differential equation

$$\frac{dy}{dx} = 1 + y^2,$$

with the boundary condition, $y(0) = 0$.

- (a) Determine $y(\pi/4)$ *analytically*.
 - (b) Show, by comparing your computer output with the exact result, that the error in determining $y(\pi/4)$ *numerically* using the second order Runge Kutta (RK2) method varies as h^2 .
6. Consider a particle of unit mass moving in a potential

$$V(x) = \frac{x^4}{4}.$$

- (a) Using your favorite method for integrating ODEs determine the period of oscillations if the maximum amplitude is (a) 0.1, (b) 1, and (c) 10.
 - (b) For a simple harmonic oscillator, does the period depend on the amplitude of oscillation? Interpret *qualitatively* the dependence of period on amplitude for the present problem.
7. For the problem in Qu. 5 show, from appropriate computer output, that the error in determining $y(\pi/4)$ using the fourth order Runge Kutta (RK4) method varies as h^4 .