PHYSICS 250
Homework 3
Due in class, Monday October 20

All problems not written out are from Kelly Ch. 3.

1. Qu. 2.
2. Qu. 3
3. Qu. 5. Also use Mathematica to plot $J_0(x)$ and the leading asyptotic behavior on the same plot, showing that they agree for large x.
4. Qu. 7.
5. Qu. 8.
   (a) Part (a). You consider the limit $t \to \infty$. This question is rather similar to the method of steepest descent except that the large parameter is imaginary.
   You may find that your results for the Fresnel integrals, $C(\infty)$ and $S(\infty)$, obtained in the previous question (and also in class) are useful.
   You should find that the integral is proportional to $t^{-1/2}$.
   (b) Part (b). The point is that if there is no point of stationary phase (i.e. no point where $\phi'(\omega) = 0$), the integral is dominated by contributions from the end points and vanishes faster, like $1/t$.
   Note: Also for the method of steepest descent, if there is no point in the range of integration where $f'(x) = 0$, the integral is dominated by contributions from the end points. However, there the differences are exponentially large (because the large parameter is real) rather than just a difference in the power.
6. Consider
$$I = \int_0^\infty e^{Nf(x)}\, dx, \text{ where } f(x) = \frac{x^2}{2} - \frac{x^4}{4}.$$
   (a) Sketch $f(x)$.
   (b) Obtain an expression for I assuming that N is large.
   (c) Consider $N = 20$ and compare your result from the last part with with the numerical value obtained from `NIntegrate`.
7. Consider the $I(g)$ defined by the following integral:
$$I(g) = \int_0^\infty \frac{e^{-t}}{1 + tg}\, dt.$$
   (a) By integrating by parts obtain a series expansion for $I(g)$, *including a remainder*.
   (b) Neglecting the remainder, show that the radius of convergence of the series is zero (the series is only asymptotic).
   (c) By allowing g to be complex show that one obtains different results for $\arg(g) \to \pi$ and $\arg(z) \to -\pi$, and show that the difference is proportional to $e^{-1/|g|}$.
   Note: you have shown that there is a branch cut along the negative real axis terminating at the origin, but the singularity is very weak (the discontinuity is exponentially small) as the origin is approached. The reason that the series is only asymptotic is that one is expanding about a point where the function is non-analytic (the origin).