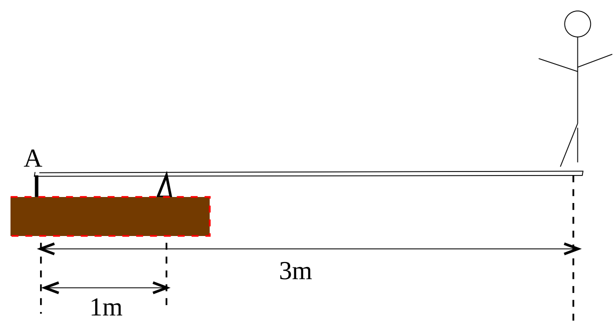
PHYSICS 5I
Final Examination
Tuesday December 9, 2009, 4:00–6:00 pm.

The exam is closed book but, if you wish, you may bring in one sheet of notes that you have prepared yourself (no photocopies).

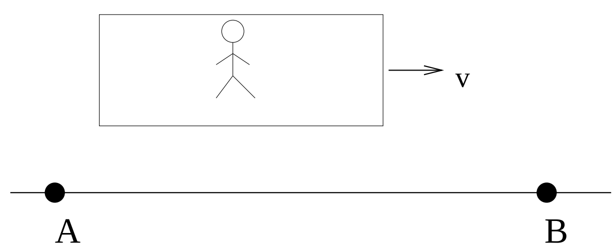
You must explain your answers.

1. [**15 points**] *Very easy; just a warm-up exercise on torque*



You are on a pirate ship and are being forced to walk the plank. Your mass is 50 kg and you are standing at the end of the plank which is 3 m long. The plank is nailed to the deck at A and rests on a support 1 m away. What is the minimum amount of force the nails at A must be able to withstand to hold the plank in place if one neglects the mass of the plank.

2. [**15 points**] *Lack of simultaneity*



Two light bulbs, A and B, are placed at rest on the x axis at $x = 0$ and $x = \ell$. In this reference frame the bulbs are turned on simultaneously. Use the Lorentz transformation to determine the time interval between when the bulbs are turned on as measured by an observer moving with speed v in the $+x$ direction. According to this observer, which light is turned on first?

3. [**20 points**] *Relativistic momentum and energy*
Two protons, each of mass m, are moving towards each with speeds of $(3/5)c$ in the laboratory frame. Determine

   (a) The momentum of each proton in the laboratory.

   (b) The total momentum of the two protons in the laboratory.

   (c) The total energy of the two protons in the laboratory.

   (d) The velocity of one proton as seen by the other.

   (e) Consider now a particle of mass $3m$ at rest, which decays into two particles each of mass m. Compute the speed of each of the resulting particles.

Note: Leave you answers in terms of m and c (i.e. don't stick in numerical values for these quantities.) Also the numerical factors should be expressed as exact fractions.

4. [**15 points**] *Outrunning a light beam.*
In class we showed that the velocity of a particle of mass m, initially at rest, subjected to a constant force F is

$$v(t) = \frac{Fct}{\sqrt{m^2c^2 + F^2t^2}} \, .$$

By integrating this expression, assuming that the particle is initially at $x = 0$, one can show that the distance traveled after time t is

$$x(t) = \frac{c}{F} \left[\sqrt{m^2c^2 + F^2t^2} - mc \right] . \tag{1}$$

Note: You are not required to derive this.

(a) Using the binomial expansion, show that, at short times, Eq. (1) reduces to the usual non-relativistic expression,

$$x(t) = \frac{1}{2}at^2, \quad \text{where the acceleration is } a = \frac{F}{m} \, .$$

(b) Using the binomial expansion, show that for very large times, i.e. in the extreme relativistic limit, that

$$x(t) = ct - \text{const.}$$

where you should determine the constant.
Note: Since a beam of light starting off at $x = -x_0$ at $t = 0$ has reached $x(t) = ct - x_0$ at time t, Eq. (1) means that an object acted on by a constant force can outrun a light beam if it starts sufficiently far ahead.

5. [**20 points**] *Numerical Integration*
We want to compute

$$I = \int_1^2 \frac{dx}{x^2} \, ,$$

i.e. the area under the curve $y = 1/x^2$ from $x = 1$ to $x = 2$. The exact result is

$$I_{\text{exact}} = \left[-\frac{1}{x} \right]_1^2 = \frac{1}{2} \, .$$

Here we are going to determine this integral *numerically.*

In class we discussed the midpoint method where

$$\int_a^b f(x)\, dx \simeq I_n \equiv h \left(f_{1/2} + f_{3/2} + \cdots f_{n-1/2} \right) \, ,$$

where $x_i = a + hi$, $f_i \equiv f(x_i)$ and we have divided the interval from a to b into n intervals each of width $h = (b-a)/n$.

Evaluate I using the midpoint rule for $n = 1, 2,$ and 4 intervals. Present your work in a table in which the columns are

$$n, \quad I_n, \quad I_n - I_{\text{exact}}, \quad n^2(I_n - I_{\text{exact}}).$$

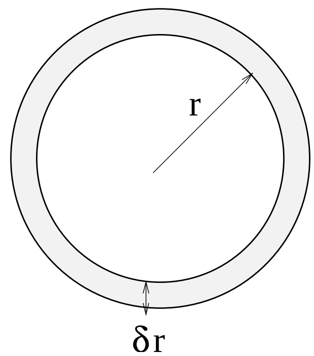
Since the error is proportional to h^2 (i.e. $\propto 1/n^2$) for small h, the numbers in the last column should approach a constant.

6. [**15 points**] *Moment of Inertia*

We will calculate the moment of inertia of a disk of radius R and mass M about an axis through its center perpendicular to the disk.

For a *ring*, where all the mass is at the same distance from the center, the moment of inertia about an axis through the center perpendicular to the ring is mr^2, where m is the mass and r is the radius of the ring.

(a) We are going to build up the disk out of many rings. Suppose we have a disc of radius r. What is the extra moment of inertia δI in adding a ring of width δr to the outside of the disk (so we now have a disk of radius $r + \delta r$). You need to know the area of a ring of radius r and thickness δr (see the figure). You could take the density (mass per unit area) to be ρ.



(b) Take the limit $\delta r \to 0$. You have now an expression for dI/dr. Integrate this expression over r from 0 to R, and rewrite your answer in terms of the total mass M rather than ρ.
Ans: $\frac{1}{2}MR^2$.