

Recent Developments in Spin Glasses

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This paper will describe some recent results in spin glasses with an emphasis on the spin glass phase transition in vector spin glasses.

§1. Introduction

Spin glasses are systems with disorder and frustration. They are very hard to treat since, for example, it is non-trivial even to find the ground state of a spin glass, and sophisticated algorithms, some borrowed from computer science, have to be used. Most theoretical work uses the Edwards-Anderson¹⁾ (EA) model,

$$\mathcal{H} = - \sum_{\langle i,j \rangle} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j, \quad (1.1)$$

in which the spins \mathbf{S}_i lie on the sites of a regular lattice, and the interactions J_{ij} , which we take to be between nearest neighbors only, are independent random variables with mean and standard deviation given by

$$[J_{ij}]_{\text{av}} = 0; \quad [J_{ij}^2]_{\text{av}}^{1/2} = J (= 1). \quad (1.2)$$

A zero mean is chosen to avoid any bias towards ferromagnetism or antiferromagnetism. In the simulations it turns out to be useful to take a Gaussian distribution for the J_{ij} . The \mathbf{S}_i are of unit length and have m -components:

$$\begin{aligned} m = 1 & \quad (\text{Ising}) \\ m = 2 & \quad (\text{XY}) \\ m = 3 & \quad (\text{Heisenberg}). \end{aligned} \quad (1.3)$$

Experimentally, there are different types of spin glasses. A classic example is an alloy with dilute magnetic atoms, e.g. Mn, in a non-magnetic host metal such as Cu, which interact with the RKKY interaction. Note that Mn is an S-state ion and so has little anisotropy. It should therefore correspond to a *Heisenberg* spin glass. There are also insulating spin glasses (which have short range interactions). An example is $\text{Fe}_{0.5}\text{Mn}_{0.5}\text{TiO}_3$, which comprises hexagonal layers. The spins align perpendicular to layers (hence it is *Ising*-like).

There is considerable evidence, some of which we will discuss, that a spin glass has a sharp thermodynamic phase transition at temperature $T = T_{SG}$, such that for $T < T_{SG}$ the spins freeze in some random-looking orientation. As $T \rightarrow T_{SG}^+$, the spin glass correlation length ξ_{SG} , which we will discuss in detail below, diverges. A

quantity which diverges, therefore, is the *spin glass susceptibility*

$$\chi_{SG} = \frac{1}{N} \sum_{\langle i,j \rangle} [\langle \mathbf{S}_i \cdot \mathbf{S}_j \rangle^2]_{av}, \quad (1.4)$$

(notice the square) which is accessible in simulations. It is also essentially *proportional to the non-linear susceptibility*, χ_{nl} , which can be measured experimentally and is defined by the coefficient of h^3 in the expansion of the magnetization m ,

$$m = \chi h - \chi_{nl} h^3 + \dots, \quad (1.5)$$

where h is the magnetic field. We expect that χ_{nl} diverges at T_{SG} like

$$\chi_{nl} \sim (T - T_{SG})^{-\gamma} \quad (1.6)$$

where γ is a critical exponent.

This divergent behavior has been seen in many experiments. We note, in particular, that of Omari et al.²⁾ on 1% Mn in Cu. They show that the dimensionless non-linear susceptibility (which is essentially *equal* to χ_{SG}) becomes very large, ($> 10^3$), and presumably diverges. A fit gives $\gamma = 3.25$.

There is a mean field solution due to Parisi^{3),4)} which following Sherrington and Kirkpatrick,⁵⁾ is the exact solution of an EA-like model with infinite range interactions. One finds a finite spin glass transition temperature T_{SG} .

Most of what we know about short range short-range (EA) models in three dimensions has come from simulations on Ising systems, which also indicate a finite T_{SG} , as we will see below. However, less is known about vector spin glass models and these will be the main focus of the rest of the talk.

While the existence of a phase transition in three-dimensions is not in serious dispute, at least for Ising spins, the nature of the equilibrium state below T_{SG} has been much more controversial. While an experimental system is not in equilibrium below T_{SG} , to develop a theory for the non-equilibrium behavior we presumably need to know the equilibrium state towards which it is trying to get to but never reaches. Two main proposals have been made for the nature of the equilibrium spin glass state:

- “Replica Symmetry Breaking (RSB), which is like the Parisi^{3),4)} mean field solution, and
- The “droplet picture” (DP) of Fisher and Huse.^{6),7)}

These differ in the nature of the large-scale, low-energy excitations, whose energy ΔE scales as

$$\Delta E \propto \ell^\theta, \quad (1.7)$$

where ℓ is the linear size of the excitation and θ is a “stiffness” exponent. RSB and DP have different predictions for θ :

- RSB, $\theta = 0$ for some excitations.
- DP, $\theta > 0$ (but small, around 0.2 for 3d Ising).

In three dimensions, numerics, which are inevitably on small lattice sizes, seem to fit best an intermediate (TNT) scenario^{8),9)} scenario. In two dimensions, where

$\theta < 0$ and consequently $T_{SG} = 0$, larger sizes can be studied and it seems that the droplet theory works, though there are significant corrections to scaling for various quantities, see e.g. Refs.^{10),11)} for recent discussions.

Another difference between the RSB and DP scenarios concerns the effects of a magnetic field. For an Ising spin glass, the DP predicts that there is no phase transition in a field, while RSB predicts that there *is* a transition, known as the “AT line”,¹²⁾ at which ξ_{SG} diverges. Below the transition line, RSB predicts that there is “replica symmetry breaking”. Convincing numerical results have been more difficult to obtain in a field than in zero field, however recently the author and H. G. Katzgraber¹³⁾ have used the “ ξ_L/L ” scaling described below and do *not* find an AT line, at least down to small fields.

In the rest of this talk I will discuss the nature of the zero field phase transition in *vector* spin glass models.

§2. Vector Spin Glasses

While there is strong evidence for a finite T_{SG} in Ising spin glasses,¹⁴⁾ many experimental systems, such as CuMn described above, are closer to an isotropic vector spin glass (\mathbf{S}_i is a vector), where the theoretical situation is less clear.

Old Monte Carlo simulations¹⁵⁾ found that T_{SG} , if it occurs at all, must be very low, and this was interpreted as being evidence for $T_{SG} = 0$. Motivated by this, Kawamura^{16)–19)} argued that $T_{SG} = 0$ but there can be a glass-like transition at $T = T_{CG}$ in the “chiralities” (i.e. vortices). This implies *spin–chirality decoupling*. However, the possibility of finite T_{SG} has been raised by various authors, e.g. Maucourt and Gempel,²⁰⁾ Akino and Kosterlitz,²¹⁾ Granato,²²⁾ Matsubara et al.,^{23),24)} and Nakamura et al.²⁵⁾

Since the most successful approach for the Ising spin glass¹⁴⁾ was finite size scaling (FSS) of the correlation length, Lee and I decided to perform analogous calculations for vector spin glasses, investigating the correlation lengths of *both* the spins and chiralities. To define chirality we follow Kawamura:^{17),18)}

$$\kappa_i^\mu = \begin{cases} \frac{1}{2\sqrt{2}} \sum_{\langle l,m \rangle} \text{sgn}(J_{lm}) \sin(\theta_l - \theta_m), & \text{XY } (\mu \perp \text{square}), \\ \mathbf{S}_{i+\hat{\mu}} \cdot \mathbf{S}_i \times \mathbf{S}_{i-\hat{\mu}}, & \text{Heisenberg}, \end{cases} \quad (2.1)$$

see Fig. 1, where for the XY case i refers to the plaquette indicated, and for the Heisenberg model, i refers to the middle of the three sites.

To determine the correlation lengths of the spins and chiralities we need to Fourier transform the appropriate correlation functions:

$$\begin{aligned} \chi_{SG}(\mathbf{k}) &= \frac{1}{N} \sum_{i,j} [\langle \mathbf{S}_i \cdot \mathbf{S}_j \rangle^2]_{\text{av}} e^{i\mathbf{k} \cdot (\mathbf{R}_i - \mathbf{R}_j)}, & (\text{spins}), \\ \chi_{CG}^\mu(\mathbf{k}) &= \frac{1}{N} \sum_{i,j} [\langle \kappa_i^\mu \kappa_j^\mu \rangle^2]_{\text{av}} e^{i\mathbf{k} \cdot (\mathbf{R}_i - \mathbf{R}_j)}, & (\text{chiralities}). \end{aligned} \quad (2.2)$$

We determine the spin glass correlation length of the finite-size system, ξ_L , from the

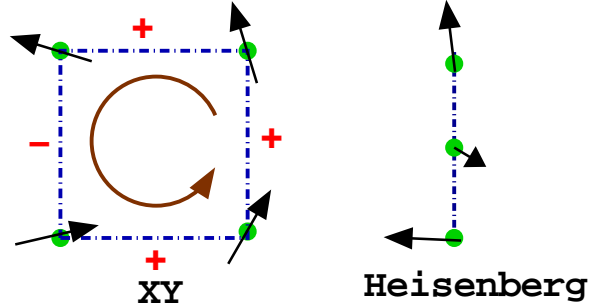


Fig. 1. An illustration of chirality for XY and Heisenberg spin glasses.

Ornstein Zernicke equation:

$$\chi_{SG}(\mathbf{k}) = \frac{\chi_{SG}(\mathbf{0})}{1 + \xi_L^2 \mathbf{k}^2 + \dots}, \quad (2.3)$$

by fitting to $\mathbf{k} = 0$ and $\mathbf{k} = \mathbf{k}_{\min} = \frac{2\pi}{L}(1, 0, 0)$. The precise formula is

$$\xi_L = \frac{1}{2 \sin(k_{\min}/2)} \left(\frac{\chi_{SG}(\mathbf{0})}{\chi_{SG}(\mathbf{k}_{\min})} - 1 \right)^{1/2}. \quad (2.4)$$

The chiral glass correlation length, $\xi_{c,L}^\mu$, is determined in an analogous way.

In order to locate the transition we use the technique of finite-size scaling (FSS). The basic assumption of FSS is that the size dependence comes from the ratio L/ξ_{bulk} where

$$\xi_{\text{bulk}} \sim (T - T_{SG})^{-\nu} \quad (2.5)$$

is the *bulk* correlation length. In particular, the *finite-size* correlation length is expected to vary as

$$\frac{\xi_L}{L} = X \left(L^{1/\nu} (T - T_{SG}) \right), \quad (2.6)$$

since ξ_L/L is dimensionless (and so has no power of L multiplying the scaling function X). Hence data for ξ_L/L for different sizes should intersect at T_{SG} and splay out below T_{SG} . Similarly, data for $\xi_{c,L}$ should intersect at T_{CG} .

§3. Results

Figure 2 shows that this works well for the Ising spin glass. This data, which is for the spin glass correlation length divided by L , shows clear intersections and hence evidence for a transition, at $T_{SG} \simeq 1.00$. Furthermore, the data splay out again on the low- T side demonstrating that there is spin glass order below T_{SG} . This is data for the Gaussian distribution. The technique of determining T_{SG} by FSS of ξ_L was first used by Ballesteros et al.¹⁴⁾ who took a distributions of bonds in which $J_{ij} = \pm 1$.

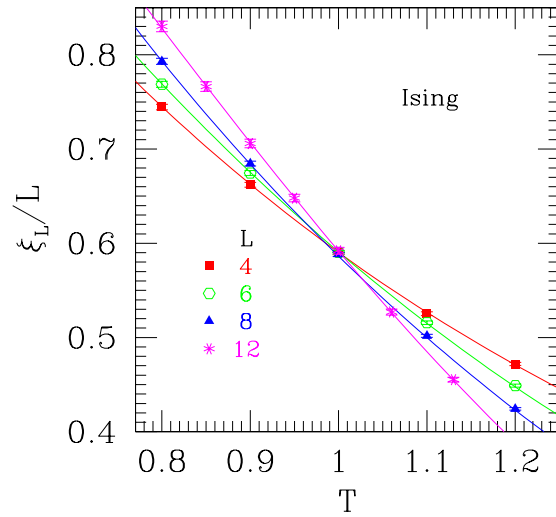


Fig. 2. Data for the correlation length of the Ising spin glass showing clear evidence for a transition at $T_{SG} \simeq 1.00$.

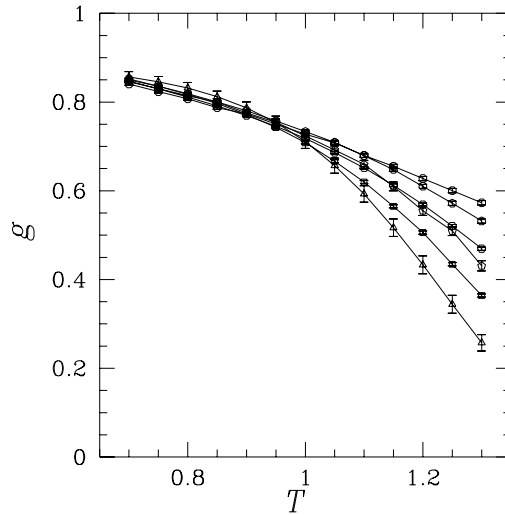


Fig. 3. Data for the Binder ratio length of the Ising spin glass with Gaussian interactions, from Marinari et al.²⁶⁾ The data merge but do not splay out strongly on the low- T side, unlike the results for the correlation length shown in Fig. 2.

Prior to the work of Ballesteros et al., determination of T_{SG} generally used the “Binder ratio”, a dimensionless ratio of the moments of the order parameter distribution which has a finite size scaling of the same form as in Eq. (2.6). However, this gives less convincing demonstration of a transition, see Fig. 3 which shows data from Marinari et al.²⁶⁾ for the Gaussian distribution. We suspect that the Binder

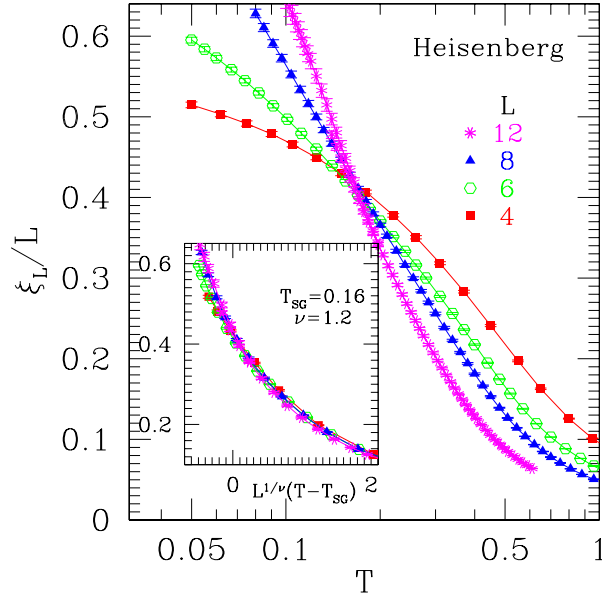


Fig. 4. Data for the spin glass correlation length of the Heisenberg spin glass divided by L (based on Ref.²⁷) with some additional data). The data intersect and splay out strongly at lower temperatures indicating a finite spin glass transition temperature. The inset is a scaling plot.

ratio is less convincing because it is constrained to be less than 1, and so has rather little “room” to splay out below T_{SG} . By contrast, ξ_L/L *diverges* as $L \rightarrow \infty$ in this region and so the splaying out is more pronounced.

We have seen that the best method for studying the transition in the Ising spin glass is FSS of the correlation length. We now apply this to the spin glass with vector spins. Similar results were obtained²⁷) for both the XY and Heisenberg models. Here, we just present results for the Heisenberg case. Figure 4 shows data for ξ_L/L . It has some additional data beyond that given in Lee and Young.²⁷) The data intersect and splay out again at low temperatures indicating a finite-temperature spin glass transition. The inset shows that the data can be collapsed reasonably according to the FSS form in Eq. (2.6) with $T_{SG} \simeq 0.16$, $\nu \simeq 1.2$.

Figure 5 shows data for the chiral correlation length. There are actually two such lengths depending upon whether the wavevector \mathbf{k}_{\min} in Eq. (2.4) is parallel or perpendicular to the line of spins shown in Fig. 1 for the Heisenberg case. The main figure in Fig. 5 shows the parallel correlation length and the inset the perpendicular correlation length. Apart from the smallest size, the data intersect pretty well. Furthermore, the transition temperature T_{CG} seems to be equal to T_{SG} , namely about 0.16.

We conclude that a *direct* study of the correlation lengths indicates that there is a single phase transition at which both spins and chiralities order in vector spin glasses.

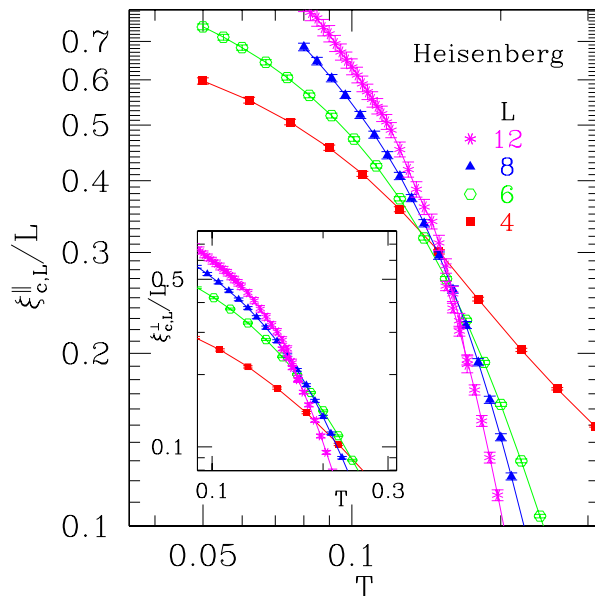


Fig. 5. Data for the chiral glass correlation length of the Heisenberg spin glass divided by L (based on Ref.²⁷⁾ with some additional data).

§4. Conclusions

We have argued that there is a finite temperature spin glass transition in vector spin glasses, even in the absence of anisotropy. By contrast Kawamura argues that there is no spin glass transition for an isotropic vector spin glass. However, in Kawamura's spin-chirality decoupling picture, any anisotropy will couple spins and chiralities leading to a spin glass transition (driven by the chiralities) at the chiral glass temperature. It is interesting, therefore, to ask what are the experimental differences between the two scenarios.

Let us denote the strength of the anisotropy by Δ , such that for $\Delta = 0$ the model is isotropic (i.e. Heisenberg-like), while for $\Delta \neq 0$ the model is anisotropic and the critical behavior is presumably Ising-like. One can *change* Δ by adding heavy impurities and see how the non-linear susceptibility diverges as a function of Δ :

$$\chi_{nl} = A(\Delta)(T - T_{SG}(\Delta))^{-\gamma}. \quad (4.1)$$

In Kawamura's picture, $T_{SG} = T_{CG}$, the chiral glass transition temperature, which is always finite except that for Δ exactly zero the spin glass transition disappears. Presumably the way this happens is that the amplitude $A(\Delta)$ vanishes for $\Delta \rightarrow 0$. Hence the different predictions are:

- This work: for $\Delta \rightarrow 0$, $A(\Delta) \rightarrow \text{const.}$, $T_{SG} \rightarrow \text{const.}$
- Kawamura: for $\Delta \rightarrow 0$, $A(\Delta) \rightarrow 0$, $T_{SG} \rightarrow \text{const.}$

Data on CuMn and AgMn alloys was obtained by Fert et al.,²⁸⁾ based on work by Vier and Schultz.²⁹⁾ The data is consistent with T_{SG} finite for $\Delta \rightarrow 0$

(or possibly $T_{SG} \rightarrow 0$ logarithmically). However, the difference between our predictions and Kawamura's is not in T_{SG} itself, which is finite even in Kawamura's scenario for any non-zero Δ , but is rather in the *amplitude* $A(\Delta)$, which we predict to be finite for $\Delta \rightarrow 0$ while Kawamura predicts that it vanishes. Unfortunately, the experiments did not appear to analyze the amplitude. We should also mention that, in our picture, the critical exponents for the isotropic model should be in a different universality class (Heisenberg) from those of the anisotropic case (Ising), and crossover between these behaviors could, in principle, be seen for weak anisotropy.

To conclude, I believe that one important question, whether or not an isotropic Heisenberg spin glass has a finite temperature spin glass transition, has been answered in the affirmative. However, the nature of the putative equilibrium state below T_{SG} , towards which the system evolves but never reaches, as well as non-equilibrium phenomena not discussed here, remain to be fully understood.

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