Mathematical Methods of Physics

Physics 116A- Winter 2018

Practice Final Examination, Total 100 Points
March 15, 2018

- Please note that this exam is for practice, and should not be construed as a model for the final paper. We are not posting solutions since these are close to what was done in class and also are close to problems in the Boas book.
- You will be allowed a calculator- without programming features.

Show details of the work and box the final results.

1. Find the disk of convergence for the series

$$\sum_{n=0}^{\infty} \frac{z^{2n}}{\sqrt{4n+1}}.$$

   Discuss the convergence on the real points on the disk.

2. Consider the following two lines that intersect at point $I(1,1,1)$ written in symmetric form:

$$\vec{r}_1 : \left(\frac{x-5/2}{-3}, \ \frac{y+1/2}{3}, \ \frac{z}{2} \right)$$
$$\vec{r}_2 : \left(\frac{x-5}{2}, \ \frac{y+5}{-3} \right) \quad z = 1$$

   a) Write the two lines in parametric form: $\vec{r} = \vec{r}_0 + \vec{A}t$.
   b) Find an equation of a plane that contains both lines.
   c) Find the *shortest* distance from the plane to point $P(1,2,3)$.

3. Evaluate

$$z = \tanh \left[\log \left(\frac{2+i}{2-i} \right) \right]$$

.

4. Consider the 2×2 matrix A given as:

$$A = \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}$$

   (a) Find eigenvalues and the left and right eigenvectors of A.

   (b) Express A as a similarity transform of a diagonal matrix.

   (c) Using the above information calculate explicitly the matrix

$$B = A^4$$

   { Hint: Avoid brute force multiplication of A }

5. Find the determinant and the inverse of the matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$

   using either the cofactor method or the row reduction method.

6. Assuming that A is a diagonal matrix in d-dimensions (i.e. with arbitrary entries $\{a_1, a_2, \ldots, a_d\}$ on the diagonal and zero everywhere else) show that

$$Det(A.B) = Det(B.A),$$

   where B is a non-diagonal matrix, also in d-dimensions.