Physics 116B- Spring 2018

Mathematical Methods 116 A

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Notes on Differential Equations-I

§Ordinary Differential equations (ODE):

Let us study one simple problem:
Consider a curve

$$y(t) = 2t^2 + 3t + 1 \tag{1}$$

§First order ODE
This is equivalent to an ODE

$$\frac{dy}{dt} = 4t + 3 \tag{2}$$

plus an initial condition (IC)

$$y(0) = 1. \tag{3}$$

Let us prove this. Integrate Eq (2)

$$\begin{aligned} \int_{y(0)}^{y(t)} dy &= \int_0^t (4t' + 3) dt' \\ y(t) - y(0) &= 2t^2 + 3t \\ \because y(t) &= y(0) + 2t^2 + 3t. \end{aligned} \tag{4}$$

Thus Eq(2) and the initial condition Eq. (3) help us recover Eq. (1) completely and exactly! If we change the value $y(0)$ given in Eq. (3), we can plug in the new value into Eq. (4) and get the new solution as well. Thus the solution of the ODE Eq. (4) contains more "juice" than what we started with in Eq. (1).

§ Another example of *an exponential process:*

$$\frac{dN(t)}{dt} = \lambda N(t). \tag{5}$$

Radioactive decay/growth, where N is the population as a function of time t !!

Solution:

$$\begin{aligned} \frac{dN}{N} &= \lambda dt \\ \int_{N_{in}}^{N(t)} \frac{dN}{N} &= \lambda \int_0^t dt \\ \log\left(\frac{N(t)}{N_{in}}\right) &= \lambda t \\ \because N(t) &= N_{in} e^{\lambda t}. \end{aligned} \tag{6}$$

Here N_{in} is defined as the initial population $N(0)$.

§Second order ODE

The same problem in Eq. (1) can also be written as a second order ODE

$$\frac{d^2y}{dt^2} = 4 \tag{7}$$

but being a second order ODE we need *two IC's*.

$$y(0) = 1, \;\; y'(0) = 3. \tag{8}$$

Let us prove it

So we get the following ideas from this example:

- ODE's express $df(x)/dx = H(x, f)$ where H is some function of x and also possibly also of $f(x)$.
- Order of the ODE is the highest derivative that occurs in the equation. (First, second,..)
- An nth order ODE needs *n- IC's* to get a unique solution. If we are not given the IC's, we can produce a general solution, which is valid for any IC's.
- the ODE's seen so far are linear, i.e. y and its derivatives occur linearly. Thus we are studying linear ODE's. However, in special cases, non-linear ODE's are equally easy to solve. An example is given below

- A partial differential equation PDE will have a function of many variables, and partial derivatives w.r.t. these variables.