An example of an exact differential equation where the function F is
F= 1/2 $\left(x^2 + y^2 + 10 \ \left(x^2 * y^2\right)\right)$

The equation is

x(1+ 10 y^2) dx + dy (1+ 10 x^2)=0

Since P = x(1+10 y^2) and Q=y (1+ 10 x^2)
we see that

P_y=20 xy

also

Q_x=20 xy $\quad = P_y$

Hence this is an exact differential equation. We can find the solution by setting
P= F_x
and Q= F_y where F can be found from integrating these equations

F= \integral dx P = 1/2 x^2(1 + 10 y^2) + A(y) (Here A is an arbitrary function of y. It is allowed since the integration over x assumes that y is a constant)

Similarly integrating Q
F= \integral dy Q= 1/2 y^2(1 + 10 x^2) + B(x)

Equating the two we get

1/2 y^2(1 + 10 x^2) + B(x)= 1/2 x^2(1 + 10 y^2) + A(y)

This equation is easily soved by choosing A(y)= 1/2 y^2 and $B(x)$=1/2 x^2

Plugging in, we get

F=1/2 $\left(x^2 + y^2 + 10\, x^2\, y^2\right)$

Summarizing, our differential equation
x(1+ 10 y^2) dx + dy (1+ 10 x^2)=0
is completely equivalent to saying that
F(x,y)= C_0,
where C_0 is an arbitrary constant and F is defined above.

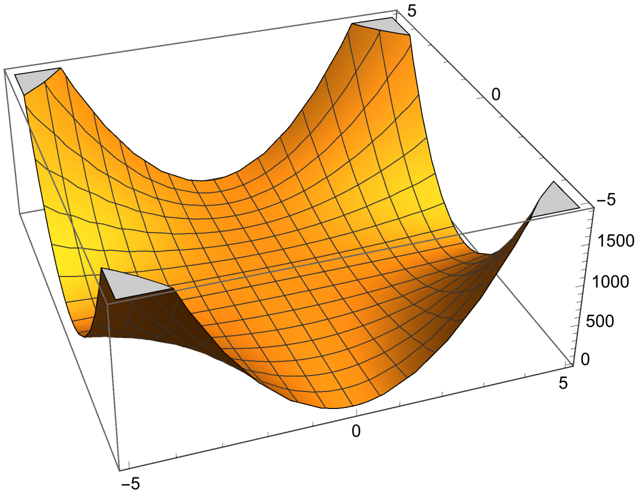
To understand the meaning of this constant, and also of F, we have spoken of F as a height of a mountain, and the value of C_0 as specifying the height at which we are seeking a (x,y) curve. Let us see this in pictures.

In[13]:=
```
f = 1 / 2 (x^2 + y^2 + 10 x^2 y^2)
```

Out[13]= $\frac{1}{2}\left(x^2 + y^2 + 10\, x^2\, y^2\right)$

In[14]:=
```
Plot3D[f, {x, -5, 5}, {y, -5, 5}]
```

Out[14]=

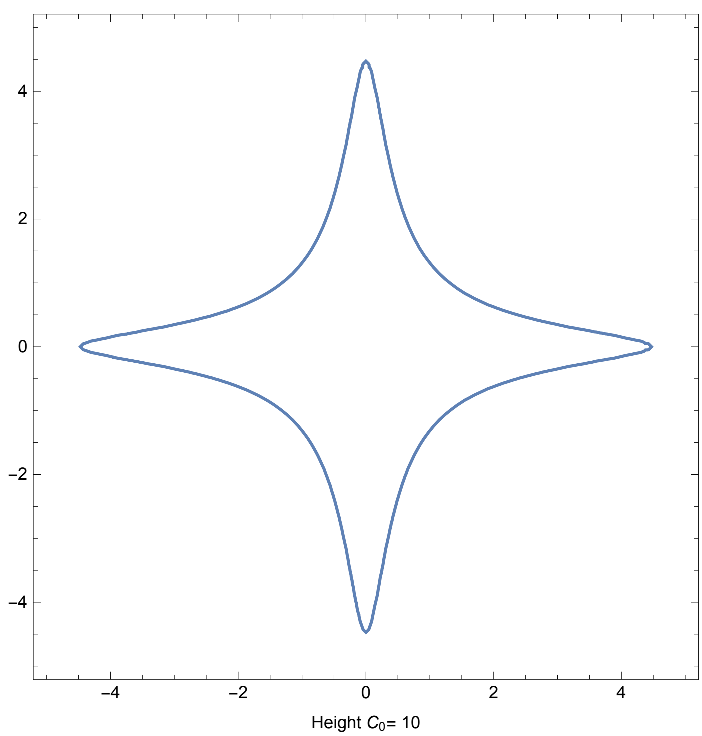


We observe that the mountain has a steep climb away from (0,0), which the bottom of the valley.

We can now specify the height C_0 as we like and discover the curves at that height.

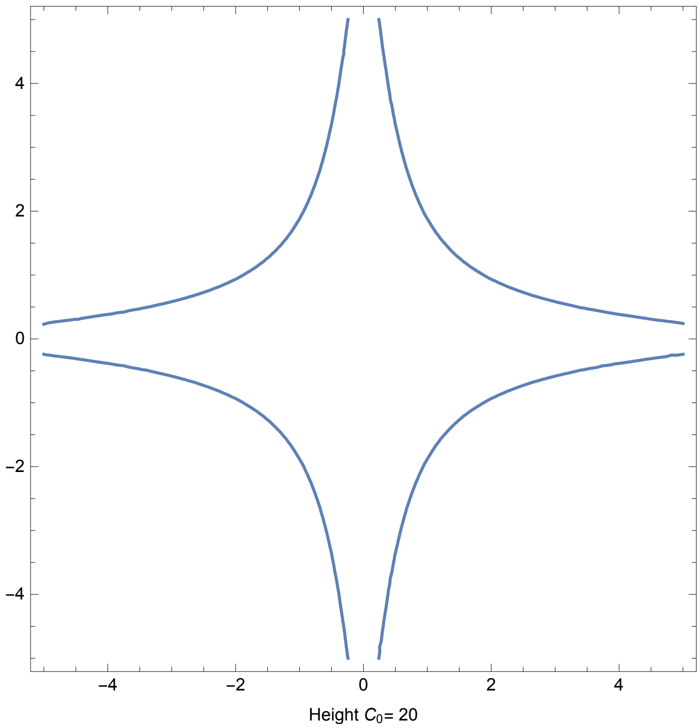
In[23]:=
```
ContourPlot[f == 10, {x, -5, 5}, {y, -5, 5}, FrameLabel → "Height C₀= 10"]
```

Out[23]=



In[20]:= `ContourPlot[f == 20, {x, -5, 5}, {y, -5, 5}, FrameLabel → "Height C₀= 20"]`

Out[20]=



In[26]:= `ContourPlot[f == 500, {x, -5, 5}, {y, -5, 5}, FrameLabel → "Height C₀= 500"]`

Out[26]=