

Table of Laplace Transforms

	$y = f(t), t > 0$ $[y = f(t) = 0, t < 0]$	$Y = L(y) = F(p) = \int_0^\infty e^{-pt} f(t) dt$	
E1	1	$\frac{1}{p}$	$\text{Re } p > 0$
E2	e^{-at}	$\frac{1}{p+a}$	$\text{Re } (p+a) > 0$
E3	$\sin at$	$\frac{a}{p^2+a^2}$	$\text{Re } p > \text{Im } a $
E4	$\cos at$	$\frac{p}{p^2+a^2}$	$\text{Re } p > \text{Im } a $
E5	$t^k, k > -1$	$\frac{k!}{p^{k+1}}$ or $\frac{\Gamma(k+1)}{p^{k+1}}$	$\text{Re } p > 0$
E6	$t^k e^{-at}, k > -1$	$\frac{k!}{(p+a)^{k+1}}$ or $\frac{\Gamma(k+1)}{(p+a)^{k+1}}$	$\text{Re } (p+a) > 0$
E7	$\frac{e^{-at} - e^{-bt}}{b-a}$	$\frac{1}{(p+a)(p+b)}$	$\text{Re } (p+a) > 0$ $\text{Re } (p+b) > 0$
E8	$\frac{ae^{-at} - be^{-bt}}{a-b}$	$\frac{p}{(p+a)(p+b)}$	$\text{Re } (p+a) > 0$ $\text{Re } (p+b) > 0$
E9	$\sinh at$	$\frac{a}{p^2-a^2}$	$\text{Re } p > \text{Re } a $
E10	$\cosh at$	$\frac{p}{p^2-a^2}$	$\text{Re } p > \text{Re } a $
E11	$t \sin at$	$\frac{2ap}{(p^2+a^2)^2}$	$\text{Re } p > \text{Im } a $
E12	$t \cos at$	$\frac{p^2-a^2}{(p^2+a^2)^2}$	$\text{Re } p > \text{Im } a $
E13	$e^{-at} \sin bt$	$\frac{b}{(p+a)^2+b^2}$	$\text{Re } (p+a) > \text{Im } b $
E14	$e^{-at} \cos bt$	$\frac{p+a}{(p+a)^2+b^2}$	$\text{Re } (p+a) > \text{Im } b $
E15	$1 - \cos at$	$\frac{a^2}{p(p^2+a^2)}$	$\text{Re } p > \text{Im } a $
E16	$at - \sin at$	$\frac{a^3}{p^2(p^2+a^2)}$	$\text{Re } p > \text{Im } a $
E17	$\sin at - at \cos at$	$\frac{2a^3}{(p^2+a^2)^2}$	$\text{Re } p > \text{Im } a $

Table of Laplace Transforms (continued)

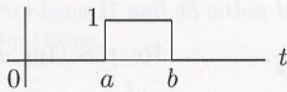
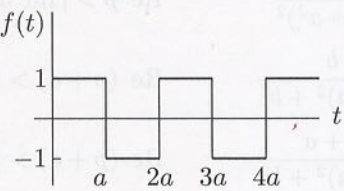
	$y = f(t), t > 0$ $[y = f(t) = 0, t < 0]$	$Y = L(y) = F(p) = \int_0^{\infty} e^{-pt} f(t) dt$	
L18	$e^{-at}(1 - at)$	$\frac{p}{(p+a)^2}$	$\operatorname{Re}(p+a) > 0$
L19	$\frac{\sin at}{t}$	$\arctan \frac{a}{p}$	$\operatorname{Re} p > \operatorname{Im} a $
L20	$\frac{1}{t} \sin at \cos bt,$ $a > 0, b > 0$	$\frac{1}{2} \left(\arctan \frac{a+b}{p} + \arctan \frac{a-b}{p} \right)$	$\operatorname{Re} p > 0$
L21	$\frac{e^{-at} - e^{-bt}}{t}$	$\ln \frac{p+b}{p+a}$	$\operatorname{Re}(p+a) > 0$ $\operatorname{Re}(p+b) > 0$
L22	$1 - \operatorname{erf} \left(\frac{a}{2\sqrt{t}} \right), a > 0$ (See Chapter 11, Section 9)	$\frac{1}{p} e^{-a\sqrt{p}}$	$\operatorname{Re} p > 0$
L23	$J_0(at)$ (See Chapter 12, Section 12)	$(p^2 + a^2)^{-1/2}$	$\operatorname{Re} p > \operatorname{Im} a ;$ or $\operatorname{Re} p \geq 0$ for real $a \neq 0$
L24	$u(t-a) = \begin{cases} 1, & t > a > 0 \\ 0, & t < a \end{cases}$ (unit step, or Heaviside function)	$\frac{1}{p} e^{-pa}$	$\operatorname{Re} p > 0$
L25	$f(t) = u(t-a) - u(t-b)$	$\frac{e^{-ap} - e^{-bp}}{p}$	All p
			
L26	$f(t)$	$\frac{1}{p} \tanh \left(\frac{1}{2} ap \right)$	$\operatorname{Re} p > 0$
			
L27	$\delta(t-a), a \geq 0$ (See Section 11)	e^{-pa}	
L28	$f(t) = \begin{cases} g(t-a), & t > a > 0 \\ 0, & t < a \end{cases}$ $= g(t-a)u(t-a)$	$e^{-pa}G(p)$ [$G(p)$ means $L(g)$.]	
L29	$e^{-at}g(t)$	$G(p+a)$	

Table of Laplace Transforms (continued)

	$y = f(t), t > 0$ $[y = f(t) = 0, t < 0]$	$Y = L(y) = F(p) = \int_0^{\infty} e^{-pt} f(t) dt$
E30	$g(at), a > 0$	$\frac{1}{a} G\left(\frac{p}{a}\right)$
E31	$\frac{g(t)}{t}$ (if integrable)	$\int_p^{\infty} G(u) du$
E32	$t^n g(t)$	$(-1)^n \frac{d^n G(p)}{dp^n}$
E33	$\int_0^t g(\tau) d\tau$	$\frac{1}{p} G(p)$
E34	$\int_0^t g(t-\tau)h(\tau) d\tau = \int_0^t g(\tau)h(t-\tau) d\tau$ (convolution of g and h , often written as $g * h$; see Section 10)	$G(p)H(p)$
E35	Transforms of derivatives of y (see Section 9):	
	$L(y') = pY - y_0$	
	$L(y'') = p^2Y - py_0 - y_0'$	
	$L(y''') = p^3Y - p^2y_0 - py_0' - y_0''$, etc.	
	$L(y^{(n)}) = p^n Y - p^{n-1}y_0 - p^{n-2}y_0' - \dots - y_0^{(n-1)}$	