Physics 155- Winter 2016

Introduction to Condensed Matter Physics

Solution of Test-2

100 points, Time 1.20 hours 8 February, 2016

1. (a) Sketch the unit cell of a centered rectangular lattice with b/a=2. ... [4]

   See sketch (1-a).

   Is this is a conventional cell or a primitive unit cell? ... [3]

   It is conventional since it has a regular shape and has more than one atom per cell.

   (b) Sketch the primitive cell of the rhombic lattice and display its relationship, if any, to above problem (i.e. Part(a)). ... [8]

   See sketch (1-b). Clearly the rhombic cell is a primitive unit cell of the centered rectangular lattice.

   (c) Compute the packing fraction of the centered square lattice. ... [20]

   See sketch (1-c). The distance α between the center of the square and the corner is clearly $a/\sqrt{2}$ from the geometry, where a is the lattice constant. The radius of the circle around each lattice point must be $\alpha/2 \sim 0.353\,a$, which is smaller than $a/2$.

   The area A' inside the circles in a unit cell is $A' = 2 \times \pi\alpha^2/4$, where the factor 2 accounts for two circles inside the square. Computing further we get $A' = \pi a^2/4$, while the area of the square is a^2. Hence the packing fraction is $\pi/4 \sim 78.5\%$.

2. (a) Using the attached graph paper, draw the parallel planes corresponding to the Miller indices {4,3}. The drawing should show that *every* shown lattice point has a plane passing through it. ... [15]

   See sketch (2-a).

   {Hint: You will need to imagine some lattice points, not shown in the portion of the lattice given, to do this task.}

   (b) Calculate the separation between the planes in units of the lattice constant using geometry and compare with the standard formula in the book for 2-dimensions namely $d = a/\sqrt{h^2 + k^2}$. ... [15].

   See sketch (2-b)

3. (a) Show that the density of states of electrons (having spin half as usual) with dispersion $\varepsilon_k = \hbar^2 k^2/(2m)$ in two dimensions is given by

$$g(\varepsilon) = A/\pi \ (m/\hbar^2), \quad \ldots [10]$$

where A is the area of the sample.

We see this as follows. Let $A = L^2$ stand for the area,

$$g(\varepsilon)d\varepsilon = 2 \times L^2/(2\pi)^2 dk_x \, dk_y = (A/\pi) k \, dk,$$

where the factor of 2 is from spin and we converted to radial coordinates $dk_x \, dk_y = k \, dk \, d\theta$ and integrated out the angle θ giving a factor of 2π. Now using $\varepsilon_k = \hbar^2 k^2/(2m)$ we get $k \, dk = m/\hbar^2 d\varepsilon$, and further simplifying we get the required answer $g(\varepsilon) = A/\pi \ (m/\hbar^2)$.

(b) Further show that the Fermi wave vector is given by

$$k_F = \sqrt{2\pi n}. \quad \ldots [10]$$

Here we integrate over k upto k_F to compute the number of particles

$$N = 2 \times A/(4\pi^2) \, 2\pi \, k_F^2/2 = A k_F^2/(2\pi).$$

Inverting we express k_F in terms of the areal density $n = N/A$ to find $k_F = \sqrt{2\pi n}$.

(c) A fluid has charged particles with density $n = 6 \times 10^{23} \ (cm)^{-3}$ traveling with a velocity of $10^5 (cm/s)$. What is the cross sectional area of the pipe so that the current carried is 1 Amp? $\quad \ldots [15]$

We write the current J in terms of the current density j as $J = j \times A$, where A is the cross sectional area. Thus the formula $A = J/(nvq_e)$ can be used to compute the area, where $J = 1amp = 1Coulomb/sec$, and q_e is the charge of the particle given as $1.6 \times 10^{-23} C$, n is the density and v is the velocity. Plugging in the given values we get

$$A \sim 10^{-6} \, cm^2.$$

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Notes

- *The charge of the fluid particles can be taken as* $q_e = -1.6 \times 10^{-23}$ *Coulomb.*
- *Please use a ruler and pencil for Problem 2 (a).*

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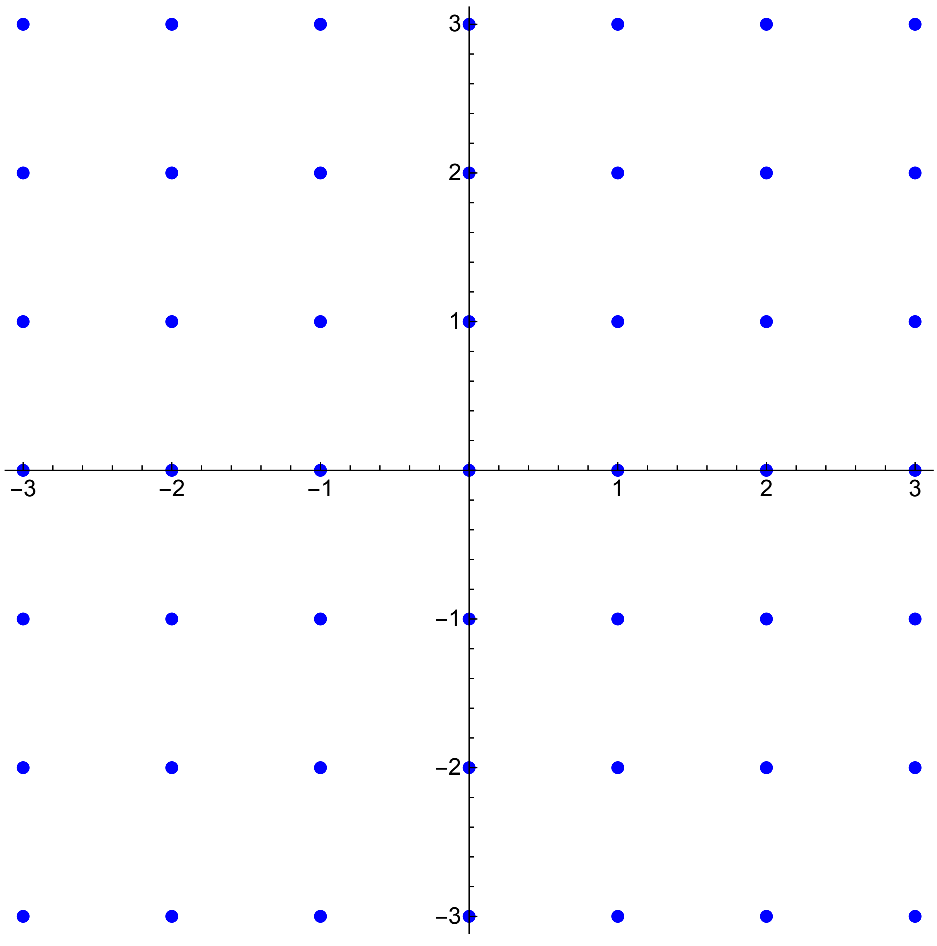
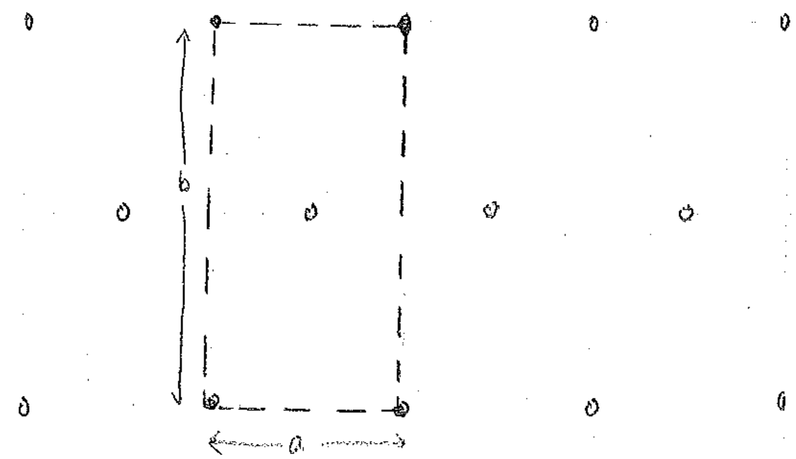


Figure 1: The square lattice

Sketch (1-a)

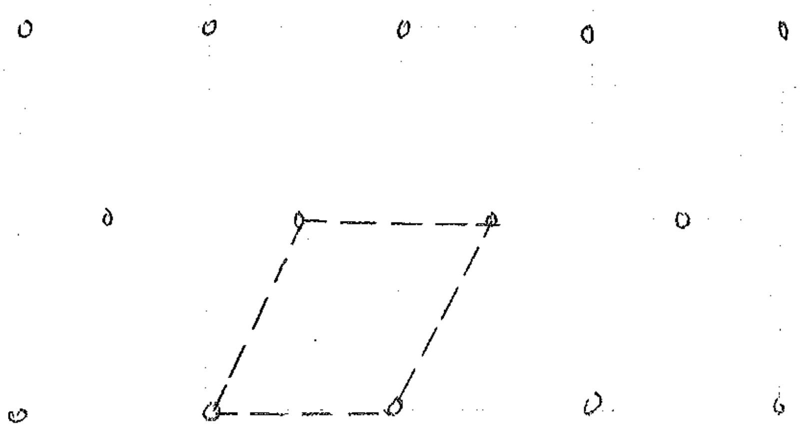


$b = 2a$

Centered rectangular lattice is conventional - #atoms/cell = 2

Sketch (1-b)

Same lattice as (1-a)



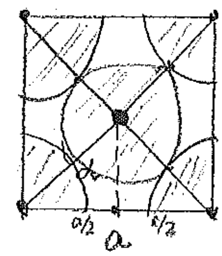
Rhombic unit cell is primitive. It is another way of looking @ the centered rectangular lattice

Sketch (1-c)

$A' =$ shaded area

$= 2 \times \pi \alpha^2/4$

$\alpha = \frac{a}{\sqrt{2}}$

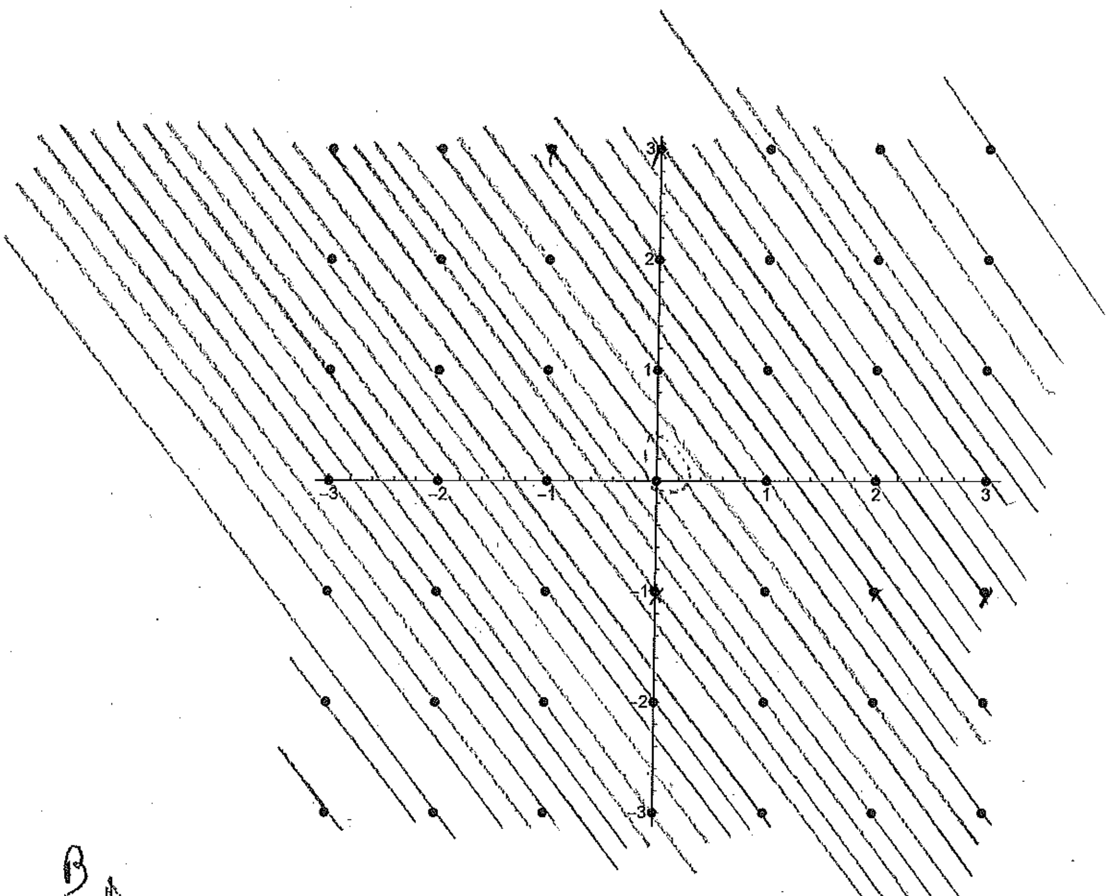


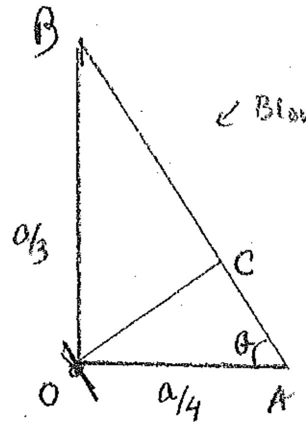
Sketch (2-a)

Miller indices $(h,k) \Rightarrow$ intercepts are $\left(\frac{a}{h}, \frac{b}{k}\right)$

Here $h=4, k=3$ $\therefore$ $\left\{\frac{a}{4}, \frac{a}{3}\right\}$ are the co-ords of the intercepts

$\therefore$ $\{3a, 4a\}$ $\therefore$ From any point I can take 3 steps to right & 4 steps to "up" and mark off the 2 points through which the Miller plane is drawn





↙ Blow up of circle near (0,0) } Sketch (2-b)

Distance between planes = "OC"

$= \text{"OA"} \sin\theta = \frac{a}{4} \times \frac{OB}{AB}$

$= \left(\frac{a}{4}\right) \times \left(\frac{a}{3}\right) \frac{1}{\sqrt{a^2/9 + a^2/16}} = \frac{a}{\sqrt{16+9}}$

$= \frac{a}{5}$!