Physics 220- Fall 2011

Theory of Many Body Physics

Rules for Ferynman Diagrams for Free energy and Greens functions

0.1 Thermodynamic Potential

We first give the rules for the thermodynamic potential $\Omega = -\frac{1}{\beta}\log Z_{GC}$ for a Fermi gas with Hamiltonian,

$$H = \sum_{r,\sigma}(\varepsilon_{r\sigma} - \mu)C^{\dagger}_{r\sigma}C_{r\sigma} + \frac{1}{2}\,\lambda \sum_{r,s,r',s',\sigma,\sigma'} \langle rs|V|r's'\rangle\; C^{\dagger}_{r\sigma}C^{\dagger}_{s\sigma'}C_{s'\sigma'}C_{r'\sigma}$$

where the electron spin is displayed and it is assumed conserved in the interaction term as in the Coulomb interaction. Here r, s are arbitrary labels and $\lambda \to 1$ at the end of the calculation.

The rules are written in terms of the free Greens function

$$G_0(r, i\omega_r) = \frac{1}{i\omega_r + \mu - \varepsilon_r} \quad \to \quad G_0(r).$$

The thermodynamic potential Ω is expanded in a formal power series in λ

$$\Omega = \sum_{n=1}^{\infty} \lambda^n \Omega^{(n)},$$

and the $\Omega^{(n)}$ is obtained using the Ferynman rules:

1. Draw all possible topologically inequivalent linked diagrams to n th order in the potential using lines for the Greens function and wavy lines for the interaction that start at a point and end up at the same point (i.e. are closed).

2. Conserve spin, frequency and if relevant momentum, at each vertex. Write labels for each Greens function line and interaction.

3. Attach a factor

$$(-1)^{n+1+n_l}\;\frac{1}{\beta}\;\frac{1}{n!}\;\frac{1}{(2\beta)^n}\;\prod_{i=1}^{n}\langle r_i s_i|V|r'_i s'_i\rangle,$$

   to this term. Here n_l is the number of closed loops that arise in the diagram.

4. Multiply this by the product of Greens functions G_0 with the correct labels.

5. Finally sum this over all state labels and frequencies. In this term there should be (n+1) independent frequencies as a check.

0.2 Greens function or self energy

In a similar way we can write the diagram rules for the self energy of the electron to nth order. These follow from Ω by taking a functional derivative as needed. Let us focus on the nth order proper self energy $\Sigma^{(n)}(r, i\omega_r)$ so that

$$\Sigma^{(n)}(r, i\omega_r) = \sum_{n=1}^{\infty} \lambda^n \Sigma^{(n)}(r, i\omega_r),$$

and the Dyson equation

$$G((r, i\omega_r) = \frac{1}{i\omega_n + \mu - \varepsilon_r - \Sigma(r, i\omega_r)},$$

or more compactly as:

$$G^{(-1)}(r, i\omega_r) = G_0^{(-1)}(r, i\omega_r) - \Sigma(r, i\omega_r).$$

We can think of r as a composite label with state (usually momentum) and frequency included.

1. To nth order in λ draw all connected topologically inequivalent self energy diagrams that have an entering and exiting label r .

2. Only proper diagrams are allowed such that no intermediate line has the external label r- since the self energy process sums over all such terms to infinite order.

3. Conserve energy spin and momentum at each vertex, attach labels to all intermediate lines and vertices.

4. Attach a factor

$$(-1)^{(n_l+n)} \frac{1}{(2\beta)^n} \prod_{i=1}^{n} \langle r_i s_i | V | r_i' s_i' \rangle.$$

5. Multiply this to the product of the G_0's with correct labels, sum over all intermediate frequencies and momenta (if relevant).