Physics 220- Fall 2011

Theory of Many Body Physics

Homework 2
6 October, 2011

1. Consider the kinetic energy operator

$$\hat{T} = (-\frac{\hbar^2}{2m}) \int d\mathbf{x}\ \psi^\dagger(\mathbf{x})\ \nabla^2\psi(\mathbf{x}),$$

and the total momentum operator

$$\hat{P} = -i\hbar \int d\mathbf{x}\ \psi^\dagger(\mathbf{x})\ \nabla\psi(\mathbf{x}),$$

and evaluate their action on the state

$$\psi\rangle = |\mathbf{x}_1\mathbf{x}_2 \ldots \mathbf{x}_N\rangle.$$

(Similar to Eq(4.79) in Coleman.)

2. Consider the field operator for a spin half electron

$$\psi^\dagger_\sigma(\mathbf{x}) = \sum_{\alpha\sigma} \phi^*_\alpha(\mathbf{x}) f^\dagger_{\alpha\sigma},$$

where I have denoted the spin component explicitly- here $\sigma = \pm 1$. Let us write the one particle state with spin and space dependence explicitly displayed as

$$|\mathbf{x}\ \xi >= \sum_{\sigma_1} \hat{\chi}_{\sigma_1}(\xi)\psi^\dagger_{\sigma_1}(\mathbf{x})|0\rangle,$$

where $\hat{\chi}_{\sigma_1}(\xi) = (1,0)$ *or* $(0,1)$ is the transposed (row) spinor, and ξ is a generalized spin coordinate. By taking overlaps, show that the one particle wave function can be expressed as

$$\psi(\mathbf{x}\ \xi) = \langle \mathbf{x}\ \xi| f^\dagger_{\alpha,\sigma}|0\rangle = \phi_\alpha(\mathbf{x})\chi_\sigma(\xi),$$

i.e. in a factor form of spin and space.

Generalize this procedure to two particle states by taking the overlap with $|\mathbf{x}_1\ \xi_1, \mathbf{x}_2\ \xi_2\rangle$ to compute the two wave functions corresponding to the Fock states

$$|a>= f^\dagger_{\alpha\uparrow} f^\dagger_{\beta\uparrow}|0\rangle,$$

and

$$|b>= f^\dagger_{\alpha\uparrow} f^\dagger_{\alpha\downarrow}|0\rangle,$$

3. Given the density operator for Fermions on a lattice

$$\rho_j = \sum_\sigma \rho_{j\sigma},$$

use the Pauli principle relations and anticommutators to evaluate the following objects to their most simplified forms:

$$\rho_j^3,$$

$$C_{j\uparrow}\rho_j^2.$$

4. Using the standard commutation relations for Fermions calculate the commutator for the Hubbard model given in class

$$H = -\sum_{i,\eta} t(\eta) C^\dagger_{i+\eta,\sigma} C_{i\sigma} + U \sum_j n_{j\uparrow} n_{j\downarrow},$$

allowing η to be arbitraty.

(a)

$$[H, C^\dagger_{j\uparrow}]$$

(b)

$$[H, \rho_j].$$

(c) Verify that the total number operator $\hat{N}$ is a constant of motion (i.e. commutes with H).

5. Calculate the 3-d Fourier transform

$$\int d\mathbf{q}\ \frac{e^2}{|r|}\ \exp i\mathbf{q}.\mathbf{r}\ \exp -\mu|r|$$

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