Physics 220- Fall 2011

Theory of Many Body Physics

Homework 3

13 October, 2011

1. a) For a tight binding model in 1,2 and 3 dimensions, write down the hopping (kinetic energy) Hamiltonian in real and Fourier space, assuming a linear chain, 2-d square lattices and a simple cubic lattice.

   b) Assuming that the electrons have spin half, find the Fermi wave vector, the Fermi energy and the (global) kinetic energy *per particle* in 1-dimension.

   c) How would you do the analogous calculation in 2-d? (It requires a numerical appproach- think it through and if possible write a simple program to do the necessary calculation).

   2) a) Writing the set of Pauli matrices

   $$u_\alpha = \{\sigma^x, \sigma^y, \sigma^z, \sigma^+, \sigma^-\},$$

   write down expressions for

   $$u_\alpha . u_\beta,$$

   for each $\alpha\beta$ pair.

   b) (Challenge problem) Using the Jordan Wigner string

   $$J[1,n] = \prod_{j=1}^{n-1}(-\sigma_j^z),$$

   show that we can alternately write

   $$J[1,n] = e^{i\pi \sum_{j=1}^{n-1} C_j^\dagger C_j},$$

   using the J W Fermions

   $$C_n^\dagger = J[1,n]\ \sigma_n^+,$$

   etc.

   c) Show that

   $$J[1,n]\sigma_j^\alpha = -\sigma_j^\alpha J[1,n],$$

   for $\{\alpha = x, y, +, -\}$, and $1 \leq j \leq n+1$, whereas

   $$J[1,n]\sigma_j^\alpha = \sigma_j^\alpha J[1,n],$$

   $N \geq j \geq n+1$.