Physics 220- Fall 2011

Theory of Many Body Physics

Homework 4

19 October, 2011

1. Starting with the Fermionic Hamiltonian treated in class:

$$H = -(J_x+J_y)\sum_{n=1}^{N}(c_{n+1}^\dagger c_n + h.c.) - -(J_x - J_y)\sum_{n=1}^{N}(c_n^\dagger c_{n+1}^\dagger + h.c.) - h\sum_{n=1}^{N}(2c_n^\dagger c_n - 1),$$

   perform the Fourier transform and verify that it can be written as

$$H = \sum_{k>0} H_k,$$

   where

$$H_k = \xi_k(n_k + n_{-k} - 1) - ib_k(c_k^\dagger c_{-k}^\dagger - h.c.)$$

   with $\xi_k = -2h - 2(J_x + J_y)$, and $b_k = 2(J_x - J_y)\sin(k)$.

2. Show that this quadratic form is diagonalized with the Valatin Bogoliubov transform

$$c_k = u_k\alpha_k + iv_k\beta_{-k}^\dagger,$$

   and

$$c_{-k} = -iv_k\alpha_k^\dagger + u_k\beta_{-k}.$$

   Find the expressions for u_k and v_k needed to achieve this. (It is permissible to use Mathematica or a similar software to help with the diagonalization).

   Show that the energy dispersion (obtained in class) is :

$$E_k = \{4(J_x - J_y)^2\sin(k)^2 + 4(h + (J_x + J_y)\cos(k))^2\}^{\frac{1}{2}}.$$

3. Find an expression for the average number of Fermions N_e by taking the derivative of the ground state energy w.r.t. h, employing the Ferynman Hellman theorem. Show that half filling corresponds to $h = 0$ when $J_x = J_y$.

   Setting $(J_x + J_y) = 1$, plot E_k versus k for $J_x - J_y = 0, .1, .2$ and $h = 0, .1, .2$.

4. Work out Coleman's Bose condensation example Sec 5.3.2 (Eq 5.62 etc).

5. Problem 5.1 page 102 Coleman. (We will discuss this in the class if you would like to, on Monday)

6. Problem 2 page 105 Coleman. (This is pretty close to our XY model treated above- so all you need to do is write out the mapping between the two models and the solution follows.)

7. Problem 3 page 106 Coleman.