Physics 220- Fall 2011

Theory of Many Body Physics

Homework 5
26 October, 2011

1. (i) Calculate $G_{0B} = (i\Omega_n - \xi_k)^{-1}$ for free Bosons with a Grand canonical energy ξ_k from the equation of motion followed by Fourier transforms.

   (ii) Let $G_{0B}(k, \tau)$ be the non interacting Bosonic Greens function in the imaginary time domain, obtained from the frequency domain object $G_{0B} = (i\Omega_n - \xi_k)^{-1}$

   $$G_{0B}(k, \tau),$$

   for positive and negative times using contour integration. Show that the analytic continuation $\tau \to it$ of this object to real (Schroedinger) time t gives the correct Greens function in time domain defined as

   $$G_{0B}(k, t) = -i < T_t(b_k(t)b^\dagger(0) > .$$

   The real time Greens function can be calculated by you from definition rather trivially using the Heisenberg equation of motion to get the time dependence explicitly.

2. Assuming Fermions, using Wicks theorem and the shorthand notation

   $$<< C_1\ C_2^\dagger >>= \frac{1}{Z}\ Tre^{-\beta H}(T_\tau(C(k_1, \tau_1)\ C^\dagger(k_2\tau_2)),$$

   evaluate the following objects in terms of the Greens function $G^0_{12} = - << C_1 C_2^\dagger >>$:

   $$<< C_1 C_2^\dagger C_4 C_3^\dagger >>,$$

   $$<< C_1 C_2 C_3 C_4^\dagger C_5^\dagger C_6^\dagger >> .$$

3. Writing out the Hubbard Hamiltonian in Fourier space, write down all the diagrams that contribute to its thermodynamic potential Ω to $O(U^2)$, and perform the frequency integrals over all internal lines expressing the results in terms of the fermi functions and unevaluated momentum summations. (There are very few terms when you use the conservation of spin index- in fact just two to this order).