Physics 220- Fall 2011

Theory of Many Body Physics

Homework 6

9 November, 2011

1. One problem where non commuting operators can be pulled apart easily is that of linear Bosons, as appears in quantum optics describing lasers. This can be illustrated by the operator identity

$$e^{za+z^*a^\dagger} = e^{za}e^{z^*a^\dagger}e^{\frac{1}{2}zz*}.$$

   Verify this identity to third order in the operators by expansion. More ambitiously you can prove this exactly using the fact that $[a, a^\dagger] = 1$, the RHS commuting with both a and $a^\dagger$. This is also known as the BCH identity after Baker Campbell and Hausdorff.

2. Verify the Coleman book calculation of (9.32) page 254 for the harmonic oscillator. This form of the displacement correlation is important when we learn electron phonon interactions.

   Verify Coleman's calculation of the "bubble susceptibility" $\chi(q, z)$ (9.44, 9.48) on pages 256-8.

3. Perform the Hubbard calculation from HW 5 using the rules given in class- you might not have done this last week since we did not frame the rules.

   For your convenience it is repeated here.

   Writing out the Hubbard Hamiltonian in Fourier space, write down all the diagrams that contribute to its thermodynamic potential Ω to $O(U^2)$, and perform the frequency integrals over all internal lines expressing the results in terms of the fermi functions and unevaluated momentum summations. (There are very few terms when you use the conservation of spin index- in fact just two to this order).

4. For the Hubbard Hamiltonian write down the expression for the self energy of an up electron to $O(U^2)$ in terms of three Greens functions- this is called the GGG theory (for obvious reasons). There are two internal frequencies in this term. Perform one of the two frequency summation using the technique learnt earlier. (We will perform the second one in class next time so it is important that you should do the first frequency sum- as a practice do each of the two frequencies- postponing the second to class).