Physics 220- Fall 2011

Theory of Many Body Physics

Homework 7

15 November, 2011

1. Using the non interacting Greens functions for Fermions and Bosons

$$G_0(i\omega_k, \vec{k}) = \frac{1}{i\omega_k - \xi_k},$$

$$D_0(i\omega_k, \vec{k}) = \frac{1}{i\Omega_k - \xi_k},$$

with the usual Matsubara frequencies ω_k and Ω_k (as defined in class) calculate

$$\frac{1}{\beta}\sum_{\omega_k}[G_0(i\omega_k, \vec{k})]^\alpha \ e^{i\omega_k 0^+},$$

and a similar expression for the Bosons, with $\alpha = 1, 2$. Here 0^+ is a positive infinitesimal.

Also calculate what happens when $\alpha = 1$ and the sign of the infinitesimal is reversed for Fermions.

2. Prove the "master identity"

$$n_B(\xi_1 - \xi_2) = \frac{f(\xi_1)(1 - f(\xi_2))}{f(\xi_2) - f(\xi_1)},$$

where n_B and f are the Bose and Fermi distribution functions.

3. Using the Lehmann representation for $G(k, \tau)$, calculate its spectral weight $\rho_G(k, \nu)$ in terms of matrix elements, where

$$G(i\omega_k, k) = \int_{-\infty}^{\infty} d\nu \ \frac{\rho_G(k, \nu)}{i\omega_k - \nu} \ d\nu.$$

Using this show that with $t > 0$

$$\langle C_k(t)C_k^\dagger(0)\rangle = \int_{-\infty}^{\infty} d\nu \ f(-\nu)\rho_G(k, \nu) \ e^{-i\nu t},$$

and with $t < 0$

$$\langle C_k^\dagger(0)C_k(t)\rangle = \int_{-\infty}^{\infty} d\nu \ f(\nu)\rho_G(k, \nu) \ e^{-i\nu t}.$$

4. At $T = 0$ and $t < 0$ calculate $\langle C_k^\dagger(0) C_k(t) \rangle$ for a model

$$\rho_G(k, \nu) = \frac{\Gamma}{(\pi\Gamma)^2 + (\nu - \xi_k)^2}.$$

What is the result when $t > 0$ in the above case?

Show that the spectral function satisfies the integration sum rule for any Γ.