Physics 210- Fall 2018

Classical and Statistical mechancis

Home Work # 2
Posted on October 22, 2018
Due in Class October 30, 2018

1. *Generalized Coordinates Example 1*

   {Comment: The first two problems are from Landau-Lifshitz Mechanics, where the part (a) of the problems are already solved. We will push ahead a bit more than they do. }

   a) Find the generalized coordinates for a coplanar double pendulum. (Solved in LL Problem 1 page 11) ...[2]

   b) Find the equations for the two coordinates ϕ_1, ϕ_2. ...[5]

   c) Comment on how you would solve these problems. (If you actually can solve them on a computer that would earn some extra credit) ...[3]

2. *Generalized Coordinates Example 2*

   a) Find the generalized coordinates for a simple pendulum of mass m_2 moving in the $x-y$ plane supported with a mass m_1 that is constrained to lie on a horizontal line along x axis. (Solved in LL Problem 2 page 11. See the figure in the book). ...[2]

   b) Write down the equations for the x and ϕ variables. ...[5]

   c) Comment on how you would solve these two equations. (If you actually can solve them on a computer that would earn some extra credit) ...[3]

3. *Lenz vector problems*

   a) For the gravitational problem $V(r) = -\frac{k}{r}$, we wrote down the Lenz vector $\vec{A} = \dot{\vec{r}} \times \vec{L} - k\frac{\vec{r}}{r}$. Using the equation of motion for $\vec{r}$ show that $\vec{A}$ is conserved. ...[5]

   b) From the above equation of motion show that $\vec{A}.\vec{r} - kr = \frac{L_z^2}{m}$, i.e is the equation of an ellipse. What is the eccentricity e in terms of A? ...[5]

c) Show that that $e^2 = 1 + 2EL_z^2/(mk^2)$ when expressed in terms of the energy E, and thus relate $|A|$ to E. . . . [10]

4. *Central field problem*

   Assuming that the central potential is given by $V(r) = -\frac{k}{r^\sigma}$, with $\sigma = 1, 1.5, 2$ and choosing suitable initial conditions and an illustrative value of the conserved energy and (non-zero) angular momentum:

   a) Compute and plot $r(t)$ versus t for a sufficiently large range of times t,

   b) Compute and plot $\phi(t)$ versus t using the above solution (from $mr^2\dot{\phi} = L_z$).

   c) Eliminate t and plot $r(t)$ versus ϕ to illustrate that the orbits are closed in the case of $\sigma = 1$ and not otherwise. In other cases show that the $r(t) - \phi(t)$ curves are space filling.

   . . . [10]

5. *Velocity dependent forces and energy conservation*

   We generalize Lagrange's equations to a more general form

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} = \frac{\partial L}{\partial q} + Q[q, \dot{q}]$$

   The case of physical interest in viscous damping has

$$Q = -k\dot{q}$$

   with $k > 0$

   a) Show that the equation of motion exhibits damping i.e. decay at long times by solving exactly the (simple) examples of $V = 0, \frac{kq^2}{2}$. (Here V is the potential energy in L). . . . [5]

   b) With energy $E \equiv \frac{m\dot{q}^2}{2} + V(q)$, show that its rate of change is negative, i.e. $dE/dt < 0$, due to damping. What does this mean physically? . . . [5]