Physics 220- Fall 2017

Theory of Many Body Physics

Commutator Notes -1

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A useful set of identities are listed here- they are frequently used to calculate various important variables in MBT. We denote the destruction operator for Fermions $a_j = f_j$ or for Bosons $a_j = b_j$. The number operator $n_j = a_j^\dagger a_j$, and $\hat{N} = \sum_j n_j$.

With $|0>$ as the vacuum state we note that

$$\begin{aligned} a_j|0> &= 0 \\ n_j|0> &= 0 \\ \hat{N}|0> &= 0 \end{aligned} \quad (1)$$

Using this we can calculate $\hat{N}Q|0>$ for any Q as $\hat{N}Q|0>= [\hat{N}, Q]|0>$. Very often the commutator is easy to calculate from the standard (anti)-commutators.

For calculating commutators we can use the identities

$$\begin{aligned} [AB, C] &= A[B, C] - [C, A]B \\ &= A\{B, C\} - \{C, A\}B. \end{aligned} \quad (2)$$

By using the elementary identity $[P, Q] = -[Q, p]$ and $[A, B]^\dagger = [B^\dagger, A^\dagger]$ we can evaluate many more relevant commutators.

Finally we should recall the telescopic identity for commutators (please check this for a few cases)

$$[A, B_1B_2B_3 \ldots B_m] = \sum_{j=1}^{m} B_1 \ldots B_{j-1}[A, B_j]B_{j+1} \ldots B_m \quad (3)$$