**Physics 220- Fall 2017**

# Theory of Many Body Physics

**Homework 1**

28 September, 2017

1. In taking Fourier series representation on the lattice we often need the following results. By performing the sum explicitly for a finite number of lattice points $N$, calculate the one dimensional delta type function $D(R_m)$ of the distance $R_m = a_0 m$:

$$D(R_m) = \frac{1}{N} \sum_{n=1,N} e^{i q_n R_m}, \qquad q_n = \frac{2\pi n}{L a_0}.$$

With $L = a_0 N$ verify the following:

(1) the periodicity

$$D(R_m) = D(R_m + L),$$

(2) Taking three typical values $N = 4, 16, 64$ plot $D(R_m)$ for $-L/2 \leq R_m \leq L/2$, and thereby convince yourself that to an excellent approximation we may treat

$$D(R_m) = \delta_{m,0}.$$

(3) Verify that the two (d=2) and three (d=3) dimensional generalization of $D(R_m)$ is respectively

$$D(\vec{R}) = \frac{1}{N^d} \sum_{\vec{q}} e^{i\vec{q}.\vec{R}},$$

where $\vec{R} = a_0\{m_1, m_2, ..m_d\}$ and $\vec{q} = \frac{2\pi}{N a_0}\{n_1, n_2, ..n_d\}$.

(This is essentially Problem 3.7.1 in Coleman)

2. Problem 3.7.3 (a) and (b) in Coleman

3. Consider two orbitals $\phi_a(r)$ and $\phi_b(r)$ and consider the "seed" wave function

$$\Phi(r_1\zeta_1, r_2\zeta_2, r_3\zeta_3) = \phi_a(r_1)\phi_a(r_2)\phi_b(r_3)\chi_\uparrow(\zeta_1)\chi_\downarrow(\zeta_2)\chi_\uparrow(\zeta_3).$$

Carry out an explicit antisymmetrization with respect to the three particle coordinates. One simple way is to construct

$$\Phi_{final} = \frac{1}{6!} \sum_P \Phi(P1, P2, P3)(-1)^{\delta_P},$$

where the sum is over the 6 permutations $(1, 2, 3)$, $(2, 1, 3)$, $(1, 3, 2)$ ,$(2, 3, 1)$, $(3, 1, 2)$, $(3, 2, 1)$ of the three coordinates. Here $\delta_P = \pm 1$ depending on the parity of the permutation (whether it involves an even or odd number of pairwise moves).