

Physics 220- Fall 2017

Theory of Many Body Physics

Homework 2

5 October, 2017

1. Coleman 4.9.1 (a,b,c)
2. Coleman 4.9. 2 (a-d)
3. **Supersymmetric oscillator.** (10 + 15 + 5 = 30 Punkte)

A supersymmetric oscillator is a system of non-interacting spinless bosons and fermions described by the Hamiltonian

$$\hat{\mathcal{H}} = \omega(\hat{b}^\dagger \hat{b} + \hat{f}^\dagger \hat{f}),$$

where \hat{b}^\dagger (\hat{b}) and \hat{f}^\dagger (\hat{f}) are respectively bosonic and fermionic creation (annihilation) operators.

- (a) Find the eigenstates and the eigenenergies of the oscillator, together with their degeneracies.
 - (b) Operators $\hat{Q} = \sqrt{\omega} \hat{b}^\dagger \hat{f}$ and $\hat{Q}^\dagger = \sqrt{\omega} \hat{b} \hat{f}^\dagger$ convert fermions to bosons and bosons to fermions, respectively. Show that these operators correspond to some symmetries of the Hamiltonian. Rewrite the Hamiltonian in terms of \hat{Q} and \hat{Q}^\dagger .
 - (c) Find the time dependence of the operators $\hat{Q}(t)$ and $\hat{Q}^\dagger(t)$ in the Heisenberg picture.
4. Reading list: Read Problem 4.79 in Coleman carefully, and reproduce his calculation.