**Physics 220- Fall 2017**

# Theory of Many Body Physics

**Homework 5**

3 November, 2017

1. Consider the Fermi gas ground state

$$|\Psi_0>=\prod_{\alpha\in\mathcal{S}} f^\dagger_{\alpha\uparrow} f^\dagger_{\alpha\downarrow}|0>,$$

where $\mathcal{S}$ is some set of $N/2$ labels- typically the lowest wave vectors in a plane wave basis.

(a) Show that

$$f_{\alpha_o\uparrow}|\Psi_0>,\ \ f^\dagger_{\alpha_o\uparrow}|\Psi_0>$$

vanish if $\alpha_0 \notin \mathcal{S}$ and $\alpha_0 \in \mathcal{S}$ respectively for the two cases.

(b) When non zero show that the states written above correspond to removing (adding) a particle with spin up from (to) the state $\alpha_0$.

(c) Explain why removing a particle with spin up (first case above) can be viewed as adding a hole to the Fermi gas.

§Comment: From now on we will use the notation $f(\alpha) \equiv 1(0)$ and $\bar{f}(\alpha) \equiv 0(1)$ for $\alpha \in \mathcal{S}$ ($\alpha \notin \mathcal{S}$) respectively. These are the usual Fermi occupation numbers at $T = 0$.

2. a) Show that

$$<\Psi_0|f^\dagger_{\beta\sigma'} f_{\alpha\sigma}|\Psi_0>=\delta_{\alpha\beta}\delta_{\sigma\sigma'},$$

provided $\alpha \in \mathcal{S}$

b) Show that

$$<\Psi_0|f_{\alpha\sigma} f^\dagger_{\beta\sigma'}|\Psi_0>=\delta_{\alpha\beta}\delta_{\sigma\sigma'},$$

provided $\alpha \notin \mathcal{S}$.

3. Show that

$$<\Psi_0|f^\dagger_{\beta\sigma_1} f^\dagger_{\delta\sigma_2} f_{\alpha\sigma_3} f_{\gamma\sigma_4}|\Psi_0>=\left(\delta_{\alpha\delta}\delta_{\beta\gamma}\delta_{\sigma_2\sigma_3}\delta_{\sigma_1\sigma_4}-\delta_{\alpha\beta}\delta_{\delta\gamma}\delta_{\sigma_2\sigma_4}\delta_{\sigma_1\sigma_3}\right) f(\alpha) f(\beta).$$

We discussed this in class on Thursday Nov 2nd.

4. Show that

$$< \Psi_0|f_{\beta\sigma_1} f_{\delta\sigma_2} f^\dagger_{\alpha\sigma_3} f^\dagger_{\gamma\sigma_4}|\Psi_0 >= (\delta_{\alpha\delta}\delta_{\beta\gamma}\delta_{\sigma_2\sigma_3}\delta_{\sigma_1\sigma_4} - \delta_{\alpha\beta}\delta_{\delta\gamma}\delta_{\sigma_2\sigma_4}\delta_{\sigma_1\sigma_3})\,\bar{f}(\alpha)\bar{f}(\beta).$$

5. Show that

$$< \Psi_0|f_{\beta\sigma_1} f\dagger_{\delta\sigma_2} f_{\alpha\sigma_3} f^\dagger_{\gamma\sigma_4}|\Psi_0 >= \delta_{\alpha\delta}\delta_{\beta\gamma}\delta_{\sigma_2\sigma_3}\delta_{\sigma_1\sigma_4} f(\alpha)\bar{f}(\beta) + \delta_{\alpha\gamma}\delta_{\beta\delta}\delta_{\sigma_3\sigma_4}\delta_{\sigma_1\sigma_2}\bar{f}(\alpha)\bar{f}(\beta).$$

6. Give a physical meaning to each of the identities (3,4,5) in terms of particles and holes.

7. In Coleman's book work out the section 5.3. In particular familiarize yourself with Eqs(5.47-5.56).

8. Coleman. Problem 5.3 (page 106) thoroughly. What is half filling?

9. Coleman. Problem5.4 (page 107-108). Graphene.