Physics 220- Fall 2017

Theory of Many Body Physics

Homework 6

10 November, 2017

1. Consider the free Fermi gas $H_0 = \sum_{\alpha\sigma}(\varepsilon_\alpha - \mu) f^\dagger_{\alpha\sigma} f_{\alpha\sigma}$ and the density matrix $\rho_0 = \frac{1}{Z} e^{-\beta H_0}$ where Z is the partition function, so that the thermal average
$$< A >_0 = \text{tr}\ \rho_0 A.$$

   a) Show that
$$< f^\dagger_{\beta\sigma'} f_{\alpha\sigma} >_0 = \delta_{\alpha\beta}\delta_{\sigma\sigma'} f(\varepsilon_\alpha - \mu),$$
   namely the Fermi function $f(x) = 1/(1 + e^{x\beta})$.

   b) Show that
$$< f_{\alpha\sigma} f^\dagger_{\beta\sigma'} >_0 = \delta_{\alpha\beta}\delta_{\sigma\sigma'} \bar{f}(\varepsilon_\alpha - \mu),$$
   where $\bar{f}(x) = 1 - f(x)$.

2. Show that
$$< f^\dagger_{\beta\sigma_1} f^\dagger_{\delta\sigma_2} f_{\alpha\sigma_3} f_{\gamma\sigma_4} >_0 = (\delta_{\alpha\delta}\delta_{\beta\gamma}\delta_{\sigma_2\sigma_3}\delta_{\sigma_1\sigma_4} - \delta_{\alpha\beta}\delta_{\delta\gamma}\delta_{\sigma_2\sigma_4}\delta_{\sigma_1\sigma_3})\, f(\alpha) f(\beta).$$
   where the Fermi function energies are $\varepsilon_\alpha - \mu$ etc. These look identical to the zero T results shown earlier. The paper by Gaudin on the website gives the idea of the proofs, it is a very elegant paper well worth the read.

3. Show that
$$< f_{\beta\sigma_1} f_{\delta\sigma_2} f^\dagger_{\alpha\sigma_3} f^\dagger_{\gamma\sigma_4} >_0 = (\delta_{\alpha\delta}\delta_{\beta\gamma}\delta_{\sigma_2\sigma_3}\delta_{\sigma_1\sigma_4} - \delta_{\alpha\beta}\delta_{\delta\gamma}\delta_{\sigma_2\sigma_4}\delta_{\sigma_1\sigma_3})\, \bar{f}(\alpha) \bar{f}(\beta).$$

4. In the Kohn Sham paper, verify equations (2.8) and (2.9) follow from (2.5)

5. In the Kohn-Sham paper verify (2.10) in detail.

6. For the free Fermi gas $H_0 = \sum_{\alpha\sigma}(\varepsilon_\alpha - \mu) f^\dagger_{\alpha\sigma} f_{\alpha\sigma}$ calculate the Greens functions in real times

$$G_0(\alpha, t) = -i < T_t f_\alpha(t) f^\dagger_\alpha(0) >_0$$

where T time orders in real time and also the Matsubara time Greens function

$$G_0(\alpha, \tau) = -i < T_\tau f_\alpha(\tau) f^\dagger_\alpha(0) >_0 \ .$$

7. Verify the antiperiodicity of $G(k, \tau)$ that we proved in class, and compute its frequency transform.