Physics 220- Fall 2017

Theory of Many Body Physics

Homework 7

18 November, 2017

1. Verify the periodicity of the Bose Greens function $B(q,\tau) = - < T(b_q(\tau)b_q^\dagger) >$ and compute its Fourier series to show

$$B(q, i\Omega_n) = \frac{1}{i\Omega_n - \omega_q}$$

with $\Omega_n = 2n\pi/\beta$.

2. Verify Eqs(9.1-9.13) in Coleman's book. (This is equivalent to AGD's treatment but a little more detailed.)

3. Work through the solved problem Example 9.1 in Coleman (page 255).

4. Calculate the time dependence $e^{\tau H_0} V e^{-\tau H_0}$ explicitly of the electron phonon coupling term

$$V = \sum g(q) c^\dagger_{k+q\sigma} c_{k\sigma}(b_q + b^\dagger_{-q}) \tag{1}$$

where

$$H_0 = \sum_{k\sigma} c^\dagger_{k\sigma} c_{k\sigma} + \sum_p \hbar\omega_p (b_p^\dagger b_p + \frac{1}{2}).$$

5. Consider the Greens function $G = - < T c_H(\tau_1) c_H^\dagger(\tau_2) >$. Show that this can be rewritten in terms of the interaction picture operators as

$$G = -\frac{1}{Z} Tr e^{-\beta H_0} \left[T_\tau S(\beta, 0) c_I(\tau_1) c_I^\dagger(\tau_2) \right]$$

with $Z = Tr e^{-\beta H_0} [T_\tau S(\beta, 0)]$. For this purpose consider the two cases $\tau_1 > \tau_2$ and $\tau_1 < \tau_2$ separately. (H and I are the Heisenberg and interaction picture operators respectively, and S is as defined in class and AGD.)