Physics 220- Fall 2017

Manybody Physics

Test-1
100 points, Time 90 mins
20 October, 2017

Your Name (In Capitals please) ....

1. Using Fermionic anticommutators show that

$$f_{\alpha\sigma} f_{\alpha\sigma} = 0,$$

where α is any single particle label, and also show that

$$n_{\alpha\sigma}^2 = n_{\alpha\sigma},$$

where $n_{\alpha\sigma} = f_{\alpha\sigma}^\dagger f_{\alpha\sigma}$. ...[10]

2. The Hubbard model for Fermions is given as:

$$H = -\sum_{ij} t_{ij} C_{i\sigma}^\dagger C_{j\sigma} + U \sum_i n_{i\uparrow} n_{\downarrow}.$$

Show that the interaction term can be written as

$$U \sum_i \left(\frac{1}{4} \rho_i^2 - (S_i^z)^2 \right),$$

where the density $\rho_i = \sum_\sigma n_{i\sigma}$ and spin density $S_i^z = \frac{1}{2} \sum_\sigma \sigma\, n_{i\sigma}$, and $\sigma = \pm 1$. ...[20]

3. For two spin 1 particles interacting with the Hamiltonian

$$H = |J| \vec{S}_i . \vec{S}_2,$$

find *all* the eigenvalues and degeneracies. ...[20]

{Comment: No need to write the eigenfunctions. }

4. Using standard Bosonic commutators for b_k and $b_k^\dagger$, show that

$$H_0 b_k^\dagger b_p^\dagger |0 >= (\varepsilon_k + \varepsilon_p) b_k^\dagger b_p^\dagger |0 >,$$

where $H_0 = \sum_\alpha \varepsilon_\alpha b_\alpha^\dagger b_\alpha$. ...[25]

5. We saw that a Majorana fermion Φ has the property $\Phi^2 = 1$. Consider three flavors of Majorana fermions Φ_x, Φ_y, Φ_z, which mutually anticommute. Thus we write

$$\{\Phi_a, \Phi_b\} = 2\delta_{ab}$$

with $a = x, y, z$. From these we construct three new objects

$$\tau_x = i\Phi_y\Phi_z,\ \tau_y = i\Phi_z\Phi_x,\ \ \tau_z = i\Phi_x\Phi_y.$$

Show that τ_a satisfy all the properties of Pauli matrices, namely

$$\tau_a\tau_b = i\varepsilon_{abc}\tau_c + \delta_{ab},$$

for any choice of a, b, c, and with ε_{abc} as the usual antisymmetric tensor $\varepsilon_{xyz} = 1 = -\varepsilon_{yxz}$ etc. ... [25]