

PHYSICS-2

Elementary Physics of Energy

Homework 7 Solutions

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1. We first need to determine how much energy is in each photon of a particular wavelength. This is:

$$E = \frac{hc}{\lambda} \text{ where } h = 6.626 \times 10^{-34} \frac{m^2 kg}{s} \text{ and } c = 3.0 \times 10^8 \frac{m}{s}$$

For the three wavelengths of 5000\AA , 1000\AA , and 10\AA , where $1\text{\AA} = 10^{-10}m$, we have, respectively:

$$E = 3.97 \times 10^{-19} J, 1.99 \times 10^{-18} J, 1.99 \times 10^{-16} J$$

Now, we set this energy equal to the heat energy ΔQ absorbed by the water (the photons are converted into heat). But, we are raising 1 gram of water by $1^\circ C$, which is defined as 1 calorie of energy. So, we convert E to calories and see how many single photons are in 1 calorie.

$$1 \text{ calorie} \left(\frac{4.184 J}{1 \text{ calorie}} \right) \left(\frac{1 \text{ photon}}{3.97 \times 10^{-19} J} \right) = \boxed{1.05 \times 10^{19} \text{ photons}}$$

Similarly, the other wavelengths require a different number of photons each, since they have a different energy:

$$\boxed{2.1 \times 10^{18} \text{ photons}}, \text{ and } \boxed{2.1 \times 10^{16} \text{ photons}}$$

2. Knowing the amount of energy per minute incident on every cm^2 , we must calculate the total amount of energy incident on Earth per second per square meter first by converting our units. Notice that we are not looking for the total energy, just the density, so we don't need to know the radius of the Earth.

$$\left(\frac{2 \text{ calories}}{\text{min} \cdot \text{cm}^2} \right) \left(\frac{1 \text{ min}}{60 \text{ s}} \right) \left(\frac{4.184 J}{1 \text{ calorie}} \right) \left(\frac{100 \text{ cm}}{1 \text{ m}} \right) \left(\frac{100 \text{ cm}}{1 \text{ m}} \right) = \frac{1394.7 J}{s \cdot m^2}$$

Note that we converted from cm to m twice, since in order to cancel cm^2 we must do so twice with the same conversion. Now we must determine how many photons of wavelength $\lambda = 5500\text{\AA}$ produce the necessary amount of energy. The energy of one such photon is given by the same formula in problem 1:

$$E_0 = \frac{hc}{\lambda} = 3.61 \times 10^{19} J$$

The number of photons N is given by

$$N = \frac{E}{E_0} = \boxed{3.86 \times 10^{24} \text{ photons}}$$

3. Each photon produces an electron, and a hole. The electrons and holes will travel in opposite directions in the circuit and have opposite charge, so the net flow of current is in the same direction and double what it would have been if we were considering only one or the other. Using the number of photons per second per square meter from problem 2, we must multiply it by 2 to account for both electrons and holes, and then multiply by the charge of each electron/hole to determine the flow of charge per second per square meter.

$$3.86 \times 10^{24} \frac{\text{photons}}{s \cdot m^2} \left(\frac{2 \text{ charges}}{1 \text{ photon}} \right) \left(\frac{1.6 \times 10^{-19} C}{1 \text{ charge}} \right) = \boxed{1.235 \times 10^6 \frac{C}{s \cdot m^2} \text{ or } \frac{A}{m^2}}$$

4. An efficiency of 15% means we only use $(0.15)(1520 \frac{Btu}{ft^2}) = 228 \frac{Btu}{ft^2}$. A Btu is defined as the energy required to heat 1 pound of water by $1^\circ F$. So we must determine the heat in Btu's necessary to heat 45 gallons of water by $10^\circ F$.

$$\Delta Q = (45 \text{ gallons of water}) \left(\frac{8 \text{ lbs}}{1 \text{ gallon of water}} \right) (10^\circ F) = 3600 \text{ Btu}$$

Dividing the total heat by the amount of energy per square foot we gain from the solar panels, we can determine how many $1ft^2$ panels we need, and convert to square meters.

$$3600 \text{ Btu} \left(\frac{1 \text{ ft}^2}{228 \text{ Btu}} \right) = 15.8 \text{ ft}^2 \left(\frac{1 \text{ m}}{3.3 \text{ ft}} \right)^2 = \boxed{1.45 \text{ m}^2}$$

Rounding up, we would need 2 panels of a square meter each to produce enough energy. Note that in converting from square feet to square meters we had to use the conversion factor twice to account for the squared unit, so the 3.3 is multiplied *twice*.

5. Using the formula for relating the temperature of a blackbody and the dominant wavelength of light being emitted, we can estimate the temperature of the blackbody. Note that the wavelength must be in micrometers, and the temperature in Kelvin.

$$\lambda = \frac{2898}{T}$$

For Yellow light, the range of wavelengths is roughly between .57 and .59 μm . This corresponds to Temperatures between $\boxed{4911.9K}$ and $\boxed{5084.2K}$.

For Blue light, the range of wavelengths is roughly between .45 and .49 μm . This corresponds to Temperatures between $\boxed{5914.3K}$ and $\boxed{6440K}$.