PHYSICS-2
Elementary Physics of Energy

Homework 3 Solutions
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1. $CH_4 + 2H_2 \rightarrow CO_2 + 2H_2O + 802kJ$ *per mole of methane.*

Solution: We see from this balanced equation that for every mole of CH_4 there is an equivalent number of moles for CO_2, as seen by their matching coefficients. Thus, having 802kJ per mole of CH_4 is the same as having 802kJ per mole of CO_2. Now we must figure out how many moles of CO_2 are in the reaction, using the amount of energy that came out. If there are 1000 Btu produced in the reaction, then we must convert this to Joules first, and then to equivalent moles of CO_2 that were produced in the reaction.

$$1000Btu(\frac{1055J}{1Btu})(\frac{1kJ}{1000J})(\frac{1\ mole\ CO_2}{802kJ}) = 1.315\ moles\ of\ CO_2$$

Now that we have the number of moles, we must use the molar mass of CO_2 to determine how much mass was produced.

$$1.315\ moles\ CO_2(\frac{44g}{1\ mole\ CO_2}) = \boxed{57.88g\ of\ CO_2}$$

To calculate the amount of methane, remember that both CH_4 and CO_2 have the same number of moles.

$$1.315\ moles\ CH_4(\frac{16g}{1\ mole\ CH_4}) = \boxed{21.05g\ of\ CH_4}$$

2. $2H_2 + O_{2]} \rightarrow 2H_2O + 484kJ$ *per mole of* O_2

Solution: This problem is almost exactly like the one above, but backwards; instead of starting with the amount of energy produced and determining the amount of reactant needed, we start with the amount of reactant and must determine the amount of energy produced. Knowing that 1 Tonne = 1000 kg, we start by determining how many moles of H_2 go into the reaction as fuel.

$$1Tonne(\frac{1000kg}{1Tonne})(\frac{1\ mole\ H_2}{2g})(\frac{1000g}{1kg}) = 5 \times 10^5\ moles\ H_2$$

According to the chemical formula, 484kJ of energy are output for every one mole of Oxygen, and therefore for every 2 moles of Hydrogen. If we divide the number of moles of Hydrogen by 2, then we have the number of moles of Oxygen in the reaction, and we can determine the energy output.

$$5 \times 10^5\ moles\ H_2(\frac{1\ mole\ O_2}{2\ moles\ H_2})(\frac{484kJ}{1\ mole\ O_2}) = \boxed{1.21 \times 10^8\ kJ}$$

Next we need to know how much oxygen is consumed. We know there are half the number of moles of oxygen as there are hydrogen, so all we must do is convert to mass using the molar mass of O_2.

$$2.5{\times}10^5\ moles\ O_2(\frac{32g}{1\ mole\ of\ O_2}) = \boxed{8 \times 10^6\ g = 8 \times 10^3\ kg = 8\ Tonnes}$$

3. *15kg of water at* $20°C$ *and 20kg of Copper at* $60°C$ *are mixed, what is the final temperature? The specific heat of copper is 0.358* $\frac{J}{g°C}$

Solution: In this problem, we have a closed system of Copper and Water exchanging heat with each other. Since the total heat of the system is not changing, we know that whatever heat the Copper gives up, the Water will gain exactly the same amount, i.e. heat is only transferred between the copper and water. This means we can say that:

$$\Delta Q_w = -\Delta Q_c$$

Now, we can write down the two equations for the heat change in both the water and the copper, and then plug both of them into this above equation to relate them to each other.

$$\Delta Q_w = c_w M_w \Delta T_w$$

$$\Delta Q_c = c_c M_c \Delta T_c$$

We know what the specific heats "c" are for both liquid water and solid copper, we know what the masses of the water and copper are, and we know what their initial temperatures are. What we don't know are their change in heat, and their final temperature, which is the same for both the water and copper. Plugging in the two heat equations into the first equation, and writing ΔT as $(T_{final} - T_{initial})$, we have:

$$c_w M_w (T_f - T_w) = -c_c M_c (T_f - T_c)$$

$$(4.186 \frac{J}{g°C})(15kg)(T_f - 20°C) = -(0.358 \frac{J}{g°C})(20kg)(T_f - 60°C)$$

We now have an algebraic equation that we can solve for T_f. Notice that both sides of the equation are multiplied by the same units $\frac{J}{g°C}$ and kg. Under normal circumstances, we would have to convert g to kg and cancel units that way, but since both sides of the equation are multiplied by the same units, we can immediately cancel them on both sides and not worry about that. *Note: Even if we did convert from g to kg, we would be doing the same conversion of multiplying by 1000 on both sides, which would cancel anyway.* Solving for T_f we find:

$$(4.186)(15)(T_f - 20°C) = -(0.358)(20)(T_f - 60°C)$$

$$62.79T_f - 1255.8°C = -7.16T_f + 429°C)$$

$$69.95T_f = 1685.4°C$$

$$\boxed{T_f = 24.1°C}$$

4. Solution: This problem is just like the last one, except this time we have changed the system slightly. We now have an unknown amount of water, but we know that the final temperature is actually $25°C$. Fortunately we have the exact same setup as before, but simply different unknown variables to solve for.

$$c_w M_w (T_f - T_w) = -c_c M_c (T_f - T_c)$$

$$(4.186 \frac{J}{g°C}) M_w (25°C - 20°C) = -(0.358 \frac{J}{g°C})(20kg)(25°C - 60°C)$$

$$4.186 M_w (5°C) = -0.358(20kg)(-35°C)$$

$$20.93 M_w = 250.6kg$$

$\boxed{M_w = 11.97kg}$

5. *Calculate the heat required in Btu to raise 1 tonne of water from* $40°F$ *to* $120°C$. *The latent heat of boiling for water is* $2.25MJ/kg$.

Solution: This problem must be done in 3 separate steps. Upon converting degrees F to degrees C, we find that the starting temperature is $4.4°C$. Since water boils at $100°C$, this means that the three steps we must do are as follows: 1) Raise the temperature from $4.4°C$ to $100°C$ with $\Delta Q_w = c_w M_w \Delta T_w$, 2) Convert liquid water to gaseous water with $\Delta Q_w = L_w M_w$, and 3) Raise the temperature from $100°C$ to $120°C$ using $\Delta Q_w = c_w M_w \Delta T_w$. We must keep in mind the different specific heats for different phases of water, $4.186\frac{J}{g°C}$ for liquid water, and $1.996\frac{J}{g°C}$ for water vapor, and we must convert to the appropriate units.

$$1)\ \Delta Q_1 = (4.186\frac{J}{g°C})(1Tonne)(100°C - 4.4°C)(\frac{1000kg}{1Tonne})(\frac{1000g}{1kg})$$

$$\Delta Q_1 = 4 \times 10^8 J$$

$$2)\ \Delta Q_2 = (2.25\frac{MJ}{kg})(1000kg)(\frac{10^6 J}{MJ})$$

$$\Delta Q_2 = 2.25 \times 10^9 J$$

$$3)\ \Delta Q_3 = (1.996\frac{J}{g°C})(1000kg)(120°C - 100°C)(\frac{1000g}{1kg})$$

$$\Delta Q_3 = 3.99 \times 10^7 J$$

$$\Delta Q_{total} = \Delta Q_1 + \Delta Q_2 + \Delta Q_3$$

$$\Delta Q_{total} = 2.69 \times 10^9 J(\frac{1Btu}{1055J}) = \boxed{2.55 \times 10^6 Btu}$$

6. *How much heat is liberated when 1 tonne of water goes from* $5°C$ *to ice at -15°C? The latent heat for melting of ice is 333 kJ/kg.*

This problem is exactly like problem 5, except instead of increasing in temperature we are decreasing, meaning heat is being lost (liberated) instead of being absorbed. Also, we have a different set of specific and latent heats for the new phases and phase transition we are working with. The only other new one is $2.108\frac{J}{g°C}$ for ice. This time the three steps are to go from $5°C to 0°C$ where the water starts to freeze, lose heat from freezing the water, then go from 0 to $-15°C$.

$$1)\ \Delta Q_1 = (4.186\frac{J}{g°C})(1Tonne)(0°C - 5°C)(\frac{1000kg}{1Tonne})(\frac{1000g}{1kg})$$

$$\Delta Q_1 = -2.093 \times 10^7 J$$

$$2)\ \Delta Q_2 = (-333\frac{kJ}{kg})(1000kg)(\frac{10^3 J}{kJ})$$

$$\Delta Q_2 = -3.33 \times 10^8 J$$

$$3)\ \Delta Q_3 = (2.108\frac{J}{g°C})(1000kg)(-15°C 0°C)(\frac{1000g}{1kg})$$

$$\Delta Q_3 = -3.162 \times 10^7 J$$

$$\Delta Q_{total} = \Delta Q_1 + \Delta Q_2 + \Delta Q_3$$

$$\Delta Q_{total} = -3.856 \times 10^8 J(\frac{1Btu}{1055J}) = \boxed{-3.65 \times 10^5 Btu}$$