PHYSICS-2
Elementary Physics of Energy

Homework 6
Due Date: May 18, 2012

This set of problems has no correspondence to those in RK, but are based on class notes and the hand out on resistances that is in the "Resources" folder. If necessary, you can look up the other books on reserve.

1. *A power plant produces 150MW power at 400V, and is connected to a transmission line with resistance 100 Ohms that delivers power to a home at 115 V. Design two transformers needed to ensure that the power dissipated in the transmission line is 2% of that produced assuming ideal transformers.*

   { *Hint: Break the problem into the various bits that were discussed in class. The meaning of "design", is that you should calculate the ratio of the transformer windings.* }*[50]*

   Let us first calculate the stepped up voltage at source V_{in} from the eqn derived in class
   $$P_{Joule} = P_{in}^2 \times R/V_{in}^2.$$
   Thus $V_{in}^2 = P_{in}^2 \times R/(.02 \times P_{in})$. We are given here $P_{in} = 150MW$ and $R = 100$Ohms, and we are told that $P_{Joule} = .02P_{in}$ since the loss is 2%. Solving for V_{in} we find $V_{in} = 866025$V. We should thus first step up the voltage from 400 to 866025 volts, and therefore the ratio of windings is $N_2/N_1 = 2165$

   The initial current (after the step up operation) is found from $I = P_{in}/V_{in} = 173.2$A. This current passes through the transmission line (i.e. a resistor) of magnitude 100 Ohms, and hence has a potential drop obtained from Ohm's law as $V_{drop} = 17320.5$Volts. The voltage at the end of the line is therefore $V_{final} = V_{in} - V_{drop} = 848705$V.

   The second transformer drops this voltage (848705) to 115 Volts, hence has a winding ratio $N_2/N_1 = 1/2165$.

2. *A pack of batteries produces a voltage of 60V in a circuit containing three resistances R_1, R_2, R_3 with magnitude 5, 3 and 1 ohms. Imagine two configurations that were not considered in class, and represent a slight challenge to the class- These are*

a) R_1, R_2 in parallel and R_3 in series with this combination.[25]

We can combine the first two resistors to an effective one $R_{eff} = R_1R_2/(R_1+R_2) = 15/8 Ohms$ as discussed in class. The current passing through the circuit I will split into two parts I_1 and I_2 across the first two $I = I_1+I_2$ and then the total will pass through the third resistor as well. To calculate the total current, let us note that the total potential drop of 60V arises from two contributions $I \times R_{eff} + I \times R_3 = I \times (2.875)$ and hence solving for I we get $I = 20.87$A.

The two currents I_1, I_2 are found using $I_1R_1 + I_2R_2 = IR_{eff}$ and $I = I_1 + I_2$. We thus find $I_1 = 7.83A$ and $I_2 = 13.04A$.

The voltage drops $V_n = I_nR_n$ and Joule power $P_n = I_n^2R_n$ are calculated from the given formulas and we find $V_1 = V_2 = 39.13,\ V_3 = 20.87$V and $P_1 = 306, P_2 = 510, P_3 = 435$W,

b) *R_1, R_2, R_3 all in parallel simultaneously, (i.e. imagine twisting three end points of each resistor into one knot for each of the two end-points).[25]*

For each configuration, calculate the current, power dissipated and voltage drop across each resistor.

We note that the current splits into three parts

$$I = I_1 + I_2 + I_3,$$

and since the end points of the resistors are bound together the three resistors experience the identical voltage drop

$$V = 60\ Volts = I_1R_2 = I_2R_2 = I_2R_3.$$

Thus the three resistors act as an effective resistance

$$R_{eff} = 1/(1/R_1 + 1/R_2 + 1/R_3) = .652\ Ohms$$

, and the current can be found as $I = 92$Amps.

The three currents can be found from $I_n = V/R_n$ withe the same V across each resistor. Thus we find $I_1 = 12, I_2 = 20, I_3 = 60$Amps.

We can now find the power in the three reistances from $P_n = V^2/R_n$ with the same $V = 60V$ across each resistor. Thus we find $P_1 = 720, P_2 = 1200, P_3 = 3600$W.