PHYSICS-2

Elementary Physics of Energy

Homework 5 Solutions

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1. The approach for this problem is to determine the total current in the circuit by using the formula $I = V/(R_1 + R_2)$ and then applying that current using $V = IR$ for each individual resistor. Remember that in series, each resistor has the full total current go through it.

$$I = \frac{60V}{5\Omega + 1\Omega} = 10A$$

$$V_1 = IR_1 = (10A)(5\Omega) = \boxed{50V}$$

$$V_2 = IR_2 = (10A)(1\Omega) = \boxed{10V}$$

The "Joule Heating" in each resistor is the dissipated Power (a.k.a. the rate of electrical energy lost as heat energy over time). We can use any of the following three formulas to calculate the Power through a particular resistor:

1) $P = \frac{V^2}{R}$

2) $P = VI$

3) $P = I^2R$

The simplest formula seems to be (2), so we will choose that one, though any of them will work. Keep in mind that these formulas only work for a particular resistor, and you must use the Voltage drop across ONLY THAT RESISTOR, not the entire circuit's voltage.

$$P_1 = V_1I = (50V)(10A) = \boxed{500W}$$

$$P_2 = V_2I = (10V)(10A) = \boxed{100W}$$

2. The power plant produces a power of 1000MW that is transmitted through the power line, with a voltage of 800,000V on the power plant end. If the solution is followed like in the lecture notes in lecture 15, then you can find the current in the wire by using $P = IV$, where V is the full voltage of the wire, or the "voltage of the circuit". The lecture then proceeds to calculate the dissipated Joule Heating power from the resistance of the line. We are asked in this problem to consider the same system and determine the voltage at the opposite end.

Given that the current is calculated to be 1250A using $P = IV$,with V being the circuit voltage given in the problem, we can simply use Ohm's Law with the resistance of the line to calculate the voltage *drop* across the resistor. Note that this voltage drop is the difference in voltage from one end of the line to the other, $V_1 - V_2$. V_1 is the voltage given to us in the problem, which is used to calculate the current, but now to find V_2 we must determine the voltage difference.

$$V_1 - V_2 = IR$$

$$800000V - V_2 = (1250A)(2.2\Omega)$$

$$V_2 = \boxed{797250V}$$

Alternatively, this problem was slightly ambiguous, and could have been interpreted that the entire amount of power was dissipated as heat. In this case, we would use $P = \dfrac{(V_1 - V_2)^2}{R}$ and solve for V_2.

$$10^9 W = \frac{(800000V - V_2)^2}{2.2\Omega}$$

$$V_2 = 800000V - \sqrt{(2.2\Omega)(10^9 W)} = \boxed{753096V}$$

To do part (b), all you must do is do the problem over again, but with $V_1 = 400000V$. The first method will give you:

$$V_2 = \boxed{497250V}$$

And the second method will give you:

$$V_2 = \boxed{453096V}$$

3. In this problem, we are given the Power and the voltage difference of a lightbulb. We can simply use version (1) of our Power equations in Problem 1 to solve for the resistance of our bulb.

$$P = \frac{V^2}{R}$$

$$100W = \frac{(115V)^2}{R}$$

$$R = \frac{(115V)^2}{100W} = \boxed{132.25\Omega}$$

Now that we know the resistance of the bulb, we are asked to find the amount of charge passing through the bulb in 5 minutes, in units of Coulombs. Current, in Amperes, is defined as the amount of charge in Coulombs flowing past per second:

$$I = \Delta q/t$$

We must find the current first, using any of the power formulas or Ohm's law.

$$I = \frac{P}{V} = \frac{100W}{115V} = .87A$$

$$I = \frac{\Delta q}{t}$$

$$.87A = \frac{\Delta q}{t}$$

$$\Delta q = (.87A)(5min)(\frac{60sec}{1min})(\frac{1C/s}{A}) = \boxed{261C}$$

If you use a different voltage of $V = 240V$ instead, then since the bulb still has the same resistance we have a power rating of

$$P = \frac{(240V)^2}{132.25\Omega} = \boxed{435.5W}$$

This would not be a very good idea, because power costs money. Having a higher voltage will increase the power consumption of our bulb by over 4 times, as you can see, which is very expensive. Of course, there is a trade-off, since you would have an increased brightness in the bulb as well, but this might also cause the bulb to be more likely to be burnt out since it would have been designed to withstand 115V.

4. This problem combines a few ideas that we have learned about. First, we must determine how much electrical power is being consumed using the known voltage and resistance of the heating unit.

$$P = \frac{V^2}{R} = \frac{(115V)^2}{11.5\Omega} = 1150W$$

This is the amount of power consumed, which we will need later when determining the cost of this setup. To heat up the water, however, there is only an 80% efficiency in converting electrical power to heat for the water, so we must multiply our result by 0.8. Then we multiply the power by 2 hours to get the total amount of energy used in that amount of time to heat the water.

$$P_{water} = 0.8(P_{el}) = 0.8(1150W) = 920W$$

$$E_{water} = 920\frac{J}{s}(2\ hours)(\frac{3600\ sec}{1\ hour}) = 6.624 \times 10^6 J = 6.624MJ$$

We now use the heat equation to determine how much mass of water can be raised by $50°C$.

$$E_{water} = \Delta Q = MC\Delta T$$

$$6.624 \times 10^6 J = M(\frac{4.184J}{g°C})(50°C)$$

$$M = 6.624 \times 10^6 J(\frac{g°C}{4.184J})(\frac{1}{50°C}) = 31663.5g = \boxed{31.6kg}$$

This is how much water is heated per day. Now, we want to determine the cost of running this setup for a whole month, assuming 12 cents per kWH.

$$P = 1150W = 1.15kW$$

$$Energy\ per\ day = P \times t = 1.15kW \times (\frac{2hours}{1day}) = \frac{2.3kWh}{1day}$$

$$Cost\ per\ month = \frac{2.3kWh}{1day}(\frac{\$0.12}{kWh})(\frac{30days}{1month}) = \boxed{\$8.28\ per\ month}$$