PHYSICS-2
Elementary Physics of Energy
Practice Midterm
To be reviewed in class on May 1 2012
100 Total Points

1. *A coal burning power plant burns coal at* 706^0C *and exhausts heat into a river with average temperature* 19^0C*. What is the* minimum possible *rate of thermal pollution (i.e. heat exhausted into the river) if the station generates 125 MW of electricity? [25]*

   Solution:

   The best possible machine for this purpose is a reversible one, i.e. a Carnot engine.

   The first calculation we need to do is to figure the efficiency of this engine. Since the operating temperatures (T_H, T_L) are given as $T_H = 706^0\ C = 979^0\ K$ and $T_L = 19^0\ C = 292^0\ K$, we get $\eta = (T_H - T_L)/T_H = 1 - 292/979 = .702$. This is the best efficiency possible (hence the lowest waste heat).

   Since the power station produces power at the rate of 125 MW, we infer that in a small time interval Δt the work done is $Q_H - Q_L = \Delta W = 125\Delta t$ MJ.

   From the definition of efficiency, we find $\eta = (Q_H - Q_L)/Q_H = 1 - Q_L/Q_H$. Hence $Q_L = (1-\eta)Q_H$, as well as $Q_H = \Delta W/\eta$. Therefore $Q_L = \Delta W(1/\eta - 1) = 125\Delta t(1/.702 - 1) = 53.1\Delta t$ MJ. This is the amount of heat discharged by the plant, in a time interval Δt, and hence the rate of pollution is 53.1 MW.

2. *A jeweller needs to melt a .5kg block of silver at* 20^0C*, in order to pour into her molds. How much heat is needed to achieve this in kJ?* [25]

   We view this in two stages: one is to heat silver to its melting temperature $T_B = 960.8^0C$ from 20^0C, using the known heat capacity of silver $C_H = .235kJ/(kg^0C)$, and the second to melt it using the latent heat of fusion (melting) $88.3kJ/kg$. The answer can be summarized in a neat formula

   $$Q = L_{melting}\ m + C\ m\ (T_B - T_{room}).$$

   Plugging in the various values, we find in units of kilo Joules $Q = 0.5\ kg \times 88.3\ kJ/kg + .235\ kJ/(kG^0C) \times 0.5\ kg \times 940.8^0C = 155kJ.$

3. Solar energy is incident on a parking lot with intensity 1000 W/m^2, and 75 % of it is absorbed. After 8 hours of exposure, how much energy per squared meter has been absorbed? Express your answer in Btu/m^2 and in calorie/m^2.

   If 50 % of the solar energy (again with intensity 1000 W/m^2) incident on a 3 m $\times$ 3 m surface for 30 minutes is used to heat up 100 kg of water, how much is the increase in the water temperature? [25]

   Answer

   First part: After 8 hours of exposure the energy absorbed per squared meter will be (remember that W=J/s)

$$0.75 \times \frac{1000 \text{ J}}{\text{s m}^2} \times \frac{3600 \text{ s}}{1 \text{ hr}} \times 8 \text{ hr} = 2.16 \times 10^7 \text{ J/m}^2 \tag{1}$$

   Convert this to Btu/m^2

$$2.16 \times 10^7 \frac{\text{J}}{\text{m}^2} \times \frac{1 \text{ Btu}}{1055 \text{ J}} = 2.05 \times 10^4 \text{ Btu/m}^2 \tag{2}$$

   In calorie/m^2

$$2.16 \times 10^7 \frac{\text{J}}{\text{m}^2} \times \frac{1 \text{ calorie}}{4.183 \text{ J}} = 5.16 \times 10^6 \text{ calorie/m}^2 \tag{3}$$

   Second part: On a surface of 3 x 3 = 9 m^2, the solar energy deposited in 30 minutes is

$$\frac{1000 \text{ J}}{\text{s m}^2} \times \frac{60 \text{ s}}{1 \text{ min}} \times 30 \text{ min} \times 9 \text{ m}^2 = 1.62 \times 10^7 \text{ J} = 1.62 \times 10^4 \text{ kJ} \tag{4}$$

   where we used 1 kJ = 1000 J. Now, 50% of this energy is used to heat up the water:

$$Q = 0.5 \times 1.62 \times 10^4 \text{ kJ} = 8.1 \times 10^3 \text{ kJ} \tag{5}$$

   To find the increase in the water temperature, use $Q = mC\Delta T$, where C is the heat capacity of water, m the mass and ΔT the increase in temperature. Solve for ΔT

$$\Delta T = \frac{Q}{mC} = \frac{8.1 \times 10^3 \text{ kJ}}{100 \text{ kg} \times 4.2 \text{ kJ/kg}^0\text{C}} = 19.3 \ ^0\text{C} \tag{6}$$

4. An ideal refrigerator takes in work at the rate of 5000 Btu/second and absorbs heat at the rate of 3000 Btu/second. What is the heat thrown into the environment in 1 hour? What is its efficiency? [15]

   What is its coefficient of performance if we use the machine as a heat pump instead?

   [10]

   Answer

   Let's call P_{input} the input power (5000 Btu/s), P_C the power absorbed from the cold chamber (3000 Btu/second) and P_H the power rejected into the atmosphere. From the First law of conservation of energy, we find

$$P_H = P_C + P_{input} = 8000 \text{ Btu/s} \tag{7}$$

   The efficiency of the refrigerator is the ratio of power absorbed to the input power i.e.

$$\eta = \frac{P_C}{P_{input}} = \frac{3}{5},$$

   and the heat rejected per hour is given by multiplying the power of heat rejection with the time in seconds

$$Q_H = P_H \times t = 8000 \frac{Btu}{sec} \times 3600 \ sec = 28.8 \times 10^6 \ Btu.$$

   When viewed as a heat pump, the coefficient of performance is

$$\text{C.O.P.} = \frac{Q_H}{W} = \frac{P_H}{P_{input}} = \frac{8000}{3000} = \frac{8}{3} \tag{8}$$

DATA

- Heat capacity of water $=4.2kJ/(kg^0C)$. Density of water $=1gm/cc$.
- Heat capacity of silver $=.235kJ/(kg^0C)$.
- Melting temp of silver $=960.8^0C$.
- Latent heat of fusion for silver $= 88.3kJ/kg$.
- 1 BTu = 1055 J. 1 calorie = 4.2 J.