Using Ohm's Law

Ohm's Law reads

$$V = IR.$$

If we know the current and the resistance, we may find the potential difference (voltage) between the ends of the piece of wire. For example, suppose a circuit element has a resistance of 50.0 Ω and carries a current of 3.5 A (a). Then the potential difference between the ends of the piece of wire is

$$V = IR = (3.5 \text{ A}) \times (50.0\ \Omega) = 175 \text{ V}.$$

A greater resistance would have led to a greater potential difference. If the resistance were 200 Ω (b), the potential difference would have been

$$V = IR = (3.5 \text{ A}) \times (200.0\ \Omega) = 700 \text{ V}.$$

A greater current for the same resistance would also have meant a greater potential difference. If $I = 10.0$ A (c), then the potential difference is

$$V = IR = (10.0 \text{ A}) \times (50.0\ \Omega) = 500 \text{ V}.$$

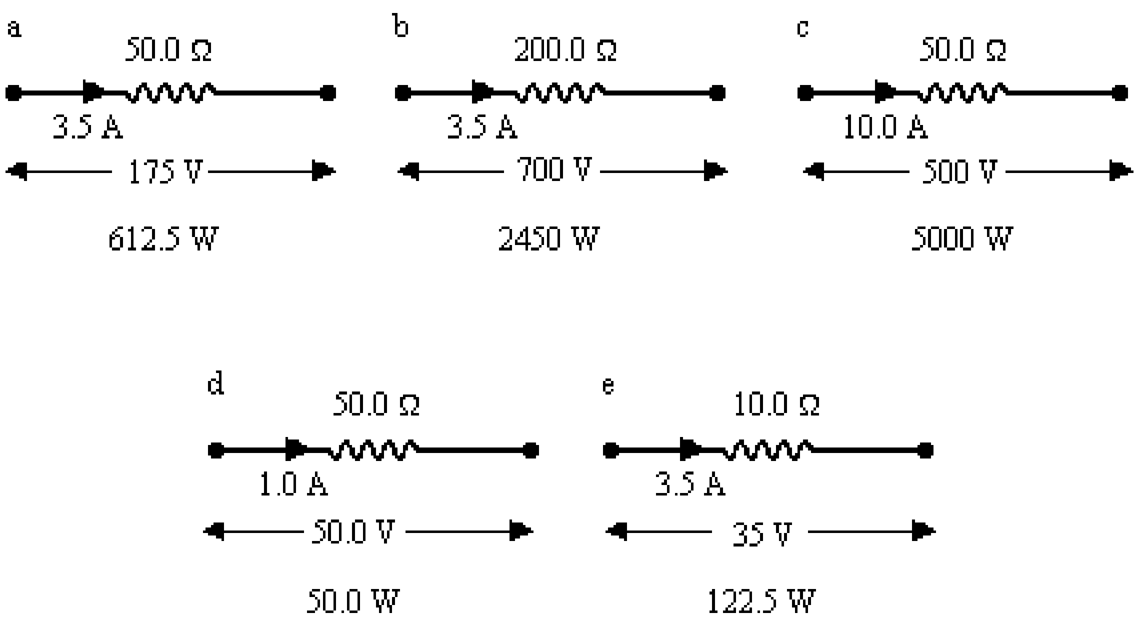

Fig. E04.3.1 Finding the voltage (potential difference) between ends of wires, and the amount of power dissipated. a. $I = 3.5$ A, $R = 50.0\ \Omega$ b. $I = 3.5$ A, $R = 200.0\ \Omega$ c. $I = 10.0$ A, $R = 50.0\ \Omega$ d. $I = 1.0$ A, $R = 50.0\ \Omega$ e. $I = 3.5$ A, $R = 10.0\ \Omega$

Similarly a smaller current, say $I = 1.0$ A (d), for the same resistance means a smaller potential difference

$$V = IR = (1.0\text{ A}) \times (50.0\ \Omega) = 50.0\text{ V}.$$

A smaller resistance R = 10.0 Ω, for the same current as in (a), 3.5 A (e) also means a smaller potential difference between the ends of the piece of wire,

$$V = IR = (3.5\text{ A}) \times (10.0\ \Omega) = 35\text{ V}.$$

Suppose we have a potential difference between two points of 100 V. If the current in the circuit element is 2.0 A, for example, we can determine the resistance to be

$$R = V/I = 100\text{ V}/2.0\text{ A} = 50.0\ \Omega.$$

A greater current flow will result if the resistance is smaller. For example, if the current in this circuit element having a potential difference of 100 V is 5.0 A, the resistance is lower:

$$R = V/I = 100\text{ V}/5.0\text{ A} = 20.0\ \Omega.$$

A smaller current flow will result when the resistance is greater. If the current in this same circuit element is 0.50 A, the resistance must have been

$$R = V/I = 100\text{ V}/0.50\text{ A} = 200.0\ \Omega.$$

Finally, if we know the potential difference between the ends of the piece of wire and the resistance, we can determine the current. Suppose a piece of wire has a potential difference between the ends of the wire of 100 V and has a resistance of 10.0 Ω. The current is

$$I = V/R = 100\text{ V}/10.0\ \Omega = 10.0\text{ A}.$$

Just as above, we can see that increasing the potential difference for a given resistance means a greater current, and increasing the resistance for a given potential difference means a smaller current. The contrary also is true: decreasing the potential difference for a given resistance means a smaller current, and decreasing the resistance for a given potential difference means a greater current.

Finding power

We know that power is given as

$$P = IV.$$

If Ohm's Law, $V = IR$, is also true, then we may find the power simply. Let us use the examples from the first paragraph above. The power used in each situation is

$$P = IV = (175\text{ V}) \times (3.5\text{ A}) = 612.5\text{ W};$$

$$P = IV = (700\text{ V}) \times (3.5\text{ A}) = 2450\text{ W};$$

$$P = IV = (500\text{ V}) \times (10.0\text{ A}) = 5000\text{ W};$$

$$P = IV = (50.0\text{ V}) \times (1.0\text{ A}) = 50.0\text{ W; and}$$

$$P = IV = (35\text{ V}) \times (3.5\text{ A}) = 122.5\text{ W, respectively}$$

If we know P and V we may determine I. Suppose in a circuit element that $V = 120$ V and $P = 1200$ W. Then

$$I = P/V = 1200\text{ W}/120\text{ V} = 10\text{ A}.$$

We may now use Ohm's Law to determine the resistance of the circuit element:

$$R = V/I = 120\text{ V}/10\text{ A} = 12\ \Omega.$$

It is obvious in this case that if R had been known, the Joule heating relation $P = I^2R$ could have been used to determine I as

$$I = \sqrt{P/R} = \sqrt{1200\text{ W}/12\ \Omega} = \sqrt{100\text{ A}^2} = 10\text{ A}.$$

As another example, consider a 10 W flashlight bulb. The two batteries that operate it produce a potential difference of 3.0 V. A current of

$$I = P/V = 10\text{ W}/3.0\text{ V} = 3.33\text{ A}.$$

will flow, the resistance may be found as

$$R = V/I = 3.0\text{ V}/3.33\text{ A} = 0.90\ \Omega.$$

Note that the Joule heat equation is consistent:

$$P = I^2R = (3.33\text{ A})^2 \times (0.90\ \Omega) = 10\text{ W}.$$

Determining Joule heating losses

We would like to be able to calculate the losses to Joule heat in various situations. Suppose a 1000 MW power plant sends its power out on a 800,000 V high voltage line. If the total resistance of the line is 2.2 Ω, what percentage of the electrical power will be lost due to Joule heating?

We may find the current in the transmission line, because we know that $P = IV$, so $I = P/V$:

$$I = \frac{P}{V} = \frac{1000 \text{ MW}}{800\,000 \text{ V}} = \frac{1\,000\,000\,000 \text{ W}}{800\,000 \text{ V}} = 1250 \text{ A}.$$

The power in Joule heating is found by $P_{\text{Joule heating}} = I^2R$,

$$P_{\text{Joule heating}} = I^2R = (1250 \text{ A})^2 \times 2.2\ \Omega = 3{,}437{,}500 \text{ W} = 3.44 \text{ MW}.$$

Hence $\frac{P_{\text{Joule heating}}}{P_{\text{transmitted}}} = \frac{3.44 \text{ MW}}{1000 \text{ MW}} = 0.344\%$ and so about one-third of a percent of the transmitted electric power is lost to Joule heating in this case.

Let's do a similar example. Suppose a 750 MW power plant sends its power out on a 364,960 V high voltage transmission line. If 1 percent of the power is lost due to Joule heating on a section of transmission line, what is the resistance of the section?

Clearly, 1% of 750 MW is 7.5 MW, so 7.5 MW of power is dissipated as Joule heat. We know the current in the line because, as mentioned above, $I = P/V$, where P is the power produced at the plant and V is the transmission line voltage. Then the current in the transmission line is

$$I = P/V = \frac{750\,000\,000 \text{ W}}{364\,960 \text{ V}} = 2055 \text{ A},$$

and the Joule heat dissipated is $P_{\text{Joule heating}} = I^2R$,

$$P_{\text{Joule heating}} = 7.5 \text{ MW} = (2055 \text{ A})^2 R,$$

so we may write

$$R = \frac{7\ 500\ 000\ \text{W}}{4\ 223\ 025\ \text{A}^2} = 1.78\ \text{W/A}^2 = 1.78\ \text{V/A} = 1.78\ \Omega.$$

This transmission line has a resistance of 1.78 Ω.

In our final examples, a 600 MW power plant sends its power out on a high voltage line. If the total resistance of a section of the line is 160 Ω, and 5% of the power is lost due to Joule heating, what is the voltage of the transmission line? We first find the loss to Joule heat, $P_{\text{Joule heating}}$ = (0.05) x (600 MW) = 30 MW. We know that $P_{\text{Joule heating}} = I^2R$, so we may determine the current in the transmission line from

$$I^2 = \frac{30\ 000\ 000\ \text{W}}{160\ \Omega} = 187{,}500\ \text{A}^2.$$

Taking the square root of this number, we find a current if I = 433 A. Since the power coming from the power plant is $P = IV$, the voltage of the transmission line must be

$$V = \frac{P}{I} = \frac{600\ 000\ 000\ \text{W}}{433\ \text{A}} = 1{,}385{,}681\ \text{V}.$$

This is not a realistic problem because there are no transmission lines operating with such high voltage. The 765,000 V transmission lines are the highest voltage lines currently in use. What if the 600 MW transmission line had a Joule heating loss of 30 MW at a voltage of 765,000 V? What would the line's resistance have to be? As in the example of the 750 MW power plant, we have

$$I = P/V = \frac{600\ 000\ 000\ \text{W}}{765\ 000\ \text{V}} = 784.3\ \text{A},$$

so $I^2 = 615{,}148\ \text{A}^2$ and

$$R = \frac{30\ 000\ 000\ \text{W}}{615\ 148\ \text{A}^2} = 48.77\ \text{W/A}^2 \approx 48.8\ \text{V/A} = 48.8\ \Omega.$$