EER, COP, and the second law efficiency for air conditioners

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It is pointed out that there is a close relationship between the energy efficiency ratio (EER) of an air conditioner unit and the coefficient of performance (COP) of its refrigeration cycle. This connection helps to bridge the gap between pure thermodynamics and practical energy-related problems. In this spirit, two other efficiency parameters, the total COP and total EER, measured relative to the energy extracted by a primary energy source (e.g., a fossil fuel), are defined. A comparison of the actual total COP (or total EER) relative to its maximum allowed value, consistent with the second law of thermodynamics, leads to an estimate for air conditioners of the recently proposed *second law efficiency*.

I. INTRODUCTION

Students of thermodynamics traditionally learn the concept of *coefficient of performance* (COP) in conjunction with the study of heat engines and refrigeration cycles.[1] The public at-large has exposure to the notion of an *energy efficiency ratio* (EER) via advertisements by electrical utility companies and/or in connection with the purchase of air conditioning equipment.[2] In this paper, (a) the close relationship between the COP and EER is delineated and (b) new efficiency parameters, the TOTAL COP and TOTAL EER, measured relative to the energy extracted from the primary energy source, are introduced. These parameters are used to discuss the recently proposed *second law efficiency*[3] within the context of refrigeration cycles.[4] Teachers can use this material to accent the relevance of thermodynamics to current, energy-related problems and to give students a more modern and practical view of the COP than appears in most textbooks. Some of the ideas presented here have been touched upon by other authors[5] and might be familiar to some teachers of physics, but they do not seem to have been exploited fully in the available literature.

II. EER AND COP

Consider a refrigeration cycle in which the working fluid (refrigerant) absorbs heat energy Q_e at the evaporator temperature T_e and releases heat $|Q_c| = -Q_c$ at the condenser temperature T_c, where $T_e < T_c$. For practical purposes in what follows, it is assumed that both Q_e and Q_c are expressed in BTU's.[6] The work done in BTU's on the refrigerant by the compressor during one complete cycle is $W = |Q_c| - Q_e$ and the coefficient of performance of the refrigeration cycle is defined as

$$\text{COP} = Q_e/W. \tag{1}$$

The maximum possible COP for a refrigerator operating between T_e and T_c is achieved in principle by a reversible Carnot refrigerator, with

$$\text{CARNOT COP} = [(T_c/T_e) - 1]^{-1} > \text{COP}. \tag{2}$$

By its very definition, the COP is a dimensionless quantity.

For a typical, real refrigeration cycle, such as in an air conditioner, the electrical energy transformed per cycle exceeds the corresponding work W done on the refrigerant due to dissipative losses in the machinery and because of the existence of a blower fan needed to circulate the air being cooled.[7] The total electrical energy transformed per cycle can be written as $(e_m^{-1})W$, where e_m is an efficiency parameter that denotes the fraction of the electrical energy delivered to the air conditioner which ultimately is transformed into mechanical work on the refrigerant. As written, the above electrical energy is expressed in BTU's and e_m is a dimensionless number satisfying $0 < e_m < 1$. Since 1 BTU = 0.293 watt hours (W h), the total electrical energy transformed per cycle is

$$\mathcal{E} = (0.293)(e_m)^{-1}W \quad (\text{W h}). \tag{3}$$

The energy efficiency ratio is defined as[8,9]

$$\text{EER} = Q_e/\mathcal{E} \quad (\text{BTU/W h}). \tag{4}$$

In what follows, it is convenient to work with *dimensionless* quantities throughout. In the Appendix, an alternative approach, using *dimensional* (as opposed to *dimensionless*) quantities such as the EER, is outlined. To begin, define a *dimensionless* EER which, in effect, is a coefficient of performance for the entire air conditioning unit. This is referred to here as the SYSTEM COP and can be written[10]

$$\text{SYSTEM COP} = (0.293)\text{EER} \quad (\textit{dimensionless}). \tag{5}$$

Combining Eqs. (1), (3), and (4), it is clear that

$$\text{COP} = (0.293)(e_m)^{-1}\,\text{EER} \quad (\textit{dimensionless}). \tag{6}$$

Equation (6) gives an explicit relationship between the COP and EER. The precise value of e_m depends upon the specific air conditioner under consideration. However, since $0 < e_m < 1$, Eqs. (5) and (6) imply

$$(0.293)\text{EER} = \text{SYSTEM COP} < \text{COP}. \tag{7}$$

The combination of inequalities (2) and (7) gives

$$\text{SYSTEM COP} < \text{COP} < \text{CARNOT COP}. \tag{8}$$

The validity of these relationships and their practical utility can be illustrated by means of a specific example. Consider a typical air conditioner unit with the following specifications[11]: cooling power = 15 628 BTU/h; T_e = 282 K (48 °F); T_c = 378 K (221 °F); EER = 6.42 BTU/W h; COP = 2.79. The double inequality (8) reduces to

$$1.88 < 2.79 < 2.94 \quad (\textit{dimensionless}). \tag{9}$$

The COP value of 2.79 is a surprising 95% of the maximum

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CARNOT COP of 2.94. However, this is a statement about the refrigeration cycle only and *not* about the performance of the entire air conditioner unit. The SYSTEM COP value of 1.88, which is a direct reflection of the latter, is only 67.4% of the COP value. That is, the efficiency parameter $e_m = 0.674$. This means that for each unit of electrical energy transformed by the air conditioner, 0.674 units go into useful compressor work on the refrigerant and 0.326 units are needed to operate the blower and to overcome a variety of dissipative losses, such as those due to fluid flow friction and mechanical losses in the compressor.

The CARNOT COP of 2.94 in (9) corresponds to an upper bound on the EER of 10 BTU/W h (see Appendix) when $T_c/T_e = 378/282 = 1.34$. It is noteworthy, however, that there do exist "high efficiency" window air conditioners, with cooling powers comparable to the one discussed here, but with EER values approaching or exceeding 10 BTU/W h.[12,13] Clearly, air conditioners with EER > 10 BTU/W h must have $T_c/T_e < 1.34$.

III. TOTAL COP AND SECOND LAW EFFICIENCY

It is apparent that the COP, which is a useful concept in the study of refrigeration cycles, is superseded in practical value by the SYSTEM COP or its *dimensional* counterpart, the EER (see Appendix). The latter two quantities reflect the energy demands of the whole air conditioner unit and are not just a property of the refrigeration cycle alone. From the point of view of energy consumption, an even more useful efficiency measure is the ratio of the heat Q_e removed per cycle from the region being cooled to the corresponding energy Q_o extracted from the primary energy source. Typically, Q_o is the heat energy obtained by the combustion of a fossil fuel, which is transformed with efficiency e_f, $0 < e_f < 1$, into electrical energy. The above ratio will be referred to here as the TOTAL COP.

$$\text{TOTAL COP} = Q_e/Q_o = (e_f e_m)\text{COP}. \quad (10)$$

The second line follows from Eq. (1) and the fact that $e_f Q_o = (e_m)^{-1} W$ (in BTU's). For the specific air conditioner above, assuming $e_f = 0.4$ (a typical value for a modern coal-fired, steam generating plant), and negligible transmission losses, Eq. (10) gives

$$\text{TOTAL COP} = 0.752 \quad (\textit{dimensionless}). \quad (11)$$

What is the maximum possible TOTAL COP consistent with the laws of thermodynamics? The second law assures that the last factor in (10), the COP, is bounded from above by the CARNOT COP = $[(T_c/T_e) - 1]^{-1}$. The efficiency e_m is clearly bounded from above by unity. If a heat engine is used to drive an electrical generator which provides power to the air conditioner, then the second law of thermodynamics mandates that e_f is bounded from above by the Carnot thermal efficiency. On the other hand, it is possible in principle to transform an amount of energy Q_o from the primary fuel source into an *equal* amount of electrical energy by purely chemical means. An example is an ideal fuel cell which operates at constant temperature and whose efficiency is not restricted by Carnot's theorem. Thus the maximum of e_f over the class of all possible systems (which use a primary fuel source to ultimately drive an air conditioner) is unity, and (10) is maximized by

$$\text{MAX. TOTAL COP} = [(T_c/T_e) - 1]^{-1}. \quad (12)$$

The second law efficiency ϵ_s involves a comparison of the actual air conditioner-electrical generator system with the most efficient one which, in principle, can accomplish the desired cooling task. Suppose it is desired to extract heat Q_e from the region being cooled during each cycle of the working fluid. Let $Q_o^{\min}$ be the *minimum* energy needed from the primary fuel source to accomplish this. This minimum is taken over the class of all possible systems. When this minimum obtains, the TOTAL COP, Eq. (10), is a maximum, as given by (12). The second law efficiency[3] can be written

$$\epsilon_s = Q_o^{\min}/Q_o = (\text{TOTAL COP})/(\text{MAX. TOTAL COP}). \quad (13)$$

An estimate of ϵ_s for the specific air conditioner under consideration can be obtained as follows. The numerator in the second line of (13) is 0.752. The denominator is given by (12), with T_c and T_e chosen as close together as possible in order to maximize it. Since dehumidification is often an essential role of an air conditioner, T_e must be substantially lower than the desired temperature of the region being cooled. For example, if the latter temperature is 75 °F (23.9 °C) and the desired relative humidity is a modest 60%, the dew-point temperature of the air is about 60 °F (15.5 °C). Accordingly, in order to obtain a realistic value for the MAX. TOTAL COP in (12), T_e is chosen to be 286 K (55 °F).[14] The condenser temperature T_c is chosen to be 311 K (100 °F), a typical maximum summer temperature. With these choices, (12) is $[(311/286) - 1]^{-1} = 11.4$ and (13) becomes

$$\epsilon_s = 0.752/11.4 = 6.6\%. \quad (14)$$

This figure, which is based upon rough numerical estimates, is close to the previously published estimate of 4.5%.[3] The roughness of (14) is evidenced by the fact that if T_c is chosen 1% smaller, leaving T_e unchanged, the MAX. TOTAL COP is increased by 14%.

If the prior discussion is limited to systems which generate electricity using heat engines, then e_f is maximized by the thermal efficiency of a reversible Carnot heat engine. If the latter operates between 922 K (1200 °F), a realistic high temperature for a superheated steam, coal-fired plant—and a low temperature of 311 K (100 °F), then e_f has a maximum possible value of $1 - (311/922) = 0.663$. The right-hand side of Eq. (12) must be replaced by $e_f[(T_c/T_e) - 1]^{-1} = (0.663)(11.4) = 7.56$ and (13) becomes $\epsilon_s < (0.752)/(7.56) \approx 10\%$. The inequality results from the fact that the class of systems over which the TOTAL COP has been maximized is a restricted one. The 10% value is quite close to the second law efficiency of 8.6% estimated by Plumb for space heating.[4] The latter estimate is an upper bound on ϵ_s since it was obtained using a restricted class of systems (heat engines) to convert primary fuel source energy to work which subsequently drives a heat pump.

IV. DISCUSSION

The TOTAL COP provides a more meaningful measure of energy usage than the COP, the SYSTEM COP or its *dimensional* counterpart, the EER. The second law efficiency illustrates how far present technology is from the limits imposed by the second law of thermodynamics.

On the other hand, it is worth pointing out that systems with Carnot efficiencies, even if they could actually be built, would not be useful. This is because such systems would entail reversible cycles, which must be carried out quasistatically. This implies that each finite cycle takes an infinite amount of time. As a consequence, the *rate* of cooling (i.e., the cooling power) for a reversible cycle is identically zero. In any real situation, finite amounts of heat flow from the ambient outdoor temperature to the region being cooled in finite time intervals. Under such circumstances, a reversible air conditioner would obviously be quite ineffective. The second law efficiency compares the economy of a real process, in terms of energy transfers, with that of an ideal process. The ideal process is the most economical process which can transfer energy between specified temperature regions, without regard for energy flow *rates*. The second law efficiency does *not* provide information on the economy of a process compared with the most economical one which accomplishes the energy transfer between specified temperature regions at a specified minimum acceptable *rate*.

Real air conditioners are designed to produce a sufficient rate of heat flow from the region being cooled to the outdoors to offset the continual heat flux into this region across its boundaries. This is done by (i) making the evaporator temperature substantially lower than the desired room temperature (this also facilitates dehumidification); (ii) blowing the air past the cooling coils at a relatively high speed; and (iii) making the condenser temperature substantially higher than the ambient outdoor temperature. This design introduces irreversible heat flows into the system which expedite the rapid transfer of heat at both the high and low temperatures. Such irreversible heat flows are costly energywise, a fact which is reflected directly in the CARNOT COP. In order to see this, recall that the air conditioner introduced in Sec. II had $T_e = 282$ K, $T_c = 378$ K, and CARNOT COP = 2.94. If instead, one were able to use $T_e = 297$ K (75 °F), the desired indoor temperature and $T_c = 311$ K (100 °F), the maximum expected outdoor temperature, the resulting CARNOT COP would be an enormous 21.2. However, as discussed above, such a design would not be conducive to the rapid flow of heat from the indoors to the outdoors.

The energy cost associated with an excessively high condenser temperature can be seen also by the realization that one could, in principle, run a heat engine between the condenser temperature and the outdoor temperature. This would recycle waste heat, generating work which could be fed back into the air conditioner, effectively raising its COP.

It is of some interest to ask if there is an optimal choice for T_c and T_e, for which the cooling capacity is maximized. An analogous question was raised in connection with heat engines by Curzon and Ahlborn.[15] They introduced a mathematical model for a heat engine which has a fixed high (combustion) temperature T_h and a fixed low (ambient outdoor) temperature T_o. They showed that there exist unique high and low temperatures, T_2 and T_1, of the working fluid for which the heat engine's power output is maximized. Curzon and Ahlborn prefaced their mathematical treatment with an argument for the existence of such a maximum power condition on the following grounds. The working fluid's temperatures T_1 and T_2 are sandwiched between the fixed temperatures T_o and T_h; i.e., $T_o \leqslant T_1 \leqslant T_2 \leqslant T_h$. When the adjustable temperatures T_1 and T_2 are chosen to be as far apart as possible, with $T_1 = T_o$ and $T_2 = T_h$, one has a reversible cycle with zero power output. When T_1 and T_2 are chosen to be as close together as possible, with $T_1 = T_2$, then zero work is performed and again, the power output is zero. Somewhere between these two extremes, the power output must be maximized. This maximum was determined analytically in Ref. 15.

For air conditioners, the situation is quite different. Here, one is dealing with the following two fixed temperatures: the desired temperature T_r, of the region being cooled and the temperature T_o, of the outdoors. These fixed temperatures are sandwiched between the adjustable evaporator temperature T_e and condenser temperature T_c; i.e., $T_e \leqslant T_r < T_o \leqslant T_c$. When T_e and T_c are chosen as close together as possible, $T_e = T_r$ and $T_c = T_o$, and one has a reversible cycle with zero cooling power. However, when $T_e < T_r$ and $T_c > T_o$, zero cooling power is not possible. Rather, the cooling power is expected to increase monotonically with T_c and with T_e^{-1}. The maximum cooling power would apparently occur for $T_e \rightarrow 0$ and T_c suitably high. Under these conditions, the CARNOT COP would approach zero. In contrast, the model heat engine of Ref. 15 achieves maximum power output at the finite thermal efficiency, $1 - (T_o/T_h)^{1/2}$.

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APPENDIX

The analyses in Secs. II and III can be carried out in *dimensional*, EER-type language. Using the fact that, $(0.293)^{-1}$ BTU/W h Eq. (1), as follows.

$$\text{DIMENSIONAL COP} = (0.293)^{-1}\text{COP} \qquad \text{(A1)}$$
$$= (0.293)^{-1}Q_e/W \quad (\text{BTU/W h}).$$

Furthermore, from Eq. (5), EER = $(0.293)^{-1}$ (SYSTEM COP). Thus multiplying each term of (8) by $(0.293)^{-1}$ gives the equivalent set of inequalities

$$\text{EER} < \text{DIMENSIONAL COP} < \text{DIMENSIONAL CARNOT COP} \quad (\text{BTU/W h}). \qquad \text{(A2)}$$

The DIMENSIONAL CARNOT COP can be thought of as a CARNOT EER. For the specific air conditioner introduced in Sec II, Eq. (A2) becomes

$$6.42 < 9.5 < 10 \quad (\text{BTU/W h}). \qquad \text{(A3)}$$

In Sec. III, the multiplication of Eq. (11) by $(0.293)^{-1}$, using the terminology, DIMENSIONAL TOTAL COP = TOTAL EER, gives

$$\text{TOTAL EER} = 2.57 \text{ BTU/W h}. \qquad \text{(A4)}$$

Similarly, the MAX. TOTAL COP value of 11.4 can be replaced by

$$\text{MAX. TOTAL EER} = 38.9\ \text{BTU/W h}, \quad \text{(A5)}$$

and the analogs of Eqs. (13) and (14) are

$$\epsilon_s = (\text{TOTAL EER})/(\text{MAX. TOTAL EER}) = 2.57/38.9 = 6.6\%. \quad \text{(A6)}$$

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[1]See, for example, A. Beiser, *Physics* (Cummings, California, 1973), p. 377; F. W. Sears, M. W. Zemansky, and H. D. Young, *University Physics* (Addison-Wesley, Reading, MA, 1976), p. 338; M. W. Zemansky, *Heat and Thermodynamics* (McGraw-Hill, New York, 1968), p. 182.

[2]The EER is explained in layman's language in Consumer Reports, **41,** 394 (July, 1976).

[3]M. H. Ross *et al., Efficient Use of Energy,* AIP Conf. Proc. No. 25 (American Institute of Physics, New York, 1975), Chap. 2. Also, C. B. Smith, *Efficient Electricity Use* (Pergamon, Elmsford, NY, 1976), Appendix C.

[4]An analysis of the second law efficiency in regard to home heating was given recently by L. R. Plumb, Am. J. Phys. **44,** 485 (1976).

[5]For example, a connection between the EER and what is referred to here as the SYSTEM COP is mentioned in Ref. 3, pp. 64, 85.

[6]The usage of the BTU here, rather than the normally preferable metric unit, the joule, allows direct contact with present day ratings of refrigeration systems. The ambitious reader, of course, can convert from British to metric units.

[7]Losses due to irreversible heat flow are discussed in Sec. IV.

[8]For older air conditioners, which do not carry an EER label, the EER can be calculated by dividing the cooling power, in BTU/h, by the power rating in watts. The latter is well approximated by the line voltage multiplied by the current.

[9]The EER units are a curious mixture of British and metric units. This reflects the historical development of energy, with heat commonly expressed in BTU's and electrical energy expressed in W h.

[10]An elementary method for measuring the SYSTEM COP of a household refrigerator was described recently by A. A. Bartlett, Am. J. Phys. **44,** 555 (1976). Bartlett calls this simply the coefficient of performance.

[11]The complete specifications for this unit were obtained from Bill Beard, Whirlpool Corp., Evansville, Indiana (private communication).

[12]*1976 Directory of Certified Room Air Conditioners* (Association of Home Appliance Manufacturers, 20 North Wacker Dr., Chicago, IL).

[13]Techniques for the achievement of higher efficiencies can be found in J. C. Moyers, "The Room Air Conditioner as an Energy Consumer," Oak Ridge National Laboratory Report, ORNL-NSF-EP-59, October, 1973.

[14]This choice makes the dehumidification capability somewhat below that for the specific unit discussed earlier, for which $T_e = 282$ K.

[15]F. L. Curzon and B. Ahlborn, Am. J. Phys. **43,** 22 (1975).