Elementary Physics of Energy

Homework 5
Due Date: May 12, 2011

1. A pack of batteries produces a voltage of 60 V in a circuit containing two resistances R_1 and R_2 in series with magnitude 5 and 1 ohms. Find the voltage drop across each resistance and the Joule heat produced in each resistor.

   Solution:

   Find the current in the system by using the relation $V_1 - V_3 = V = I(R_1 + R_2)$, given in lecture. Since $V = 60$ Volts and $R_1 + R_2 = 6$ Ohms, then $I = V/(R_1 + R_2) = 10$ Amps. Now the voltage drop across each resistor can be found from $V = I \times R$, so it is 50 V and 10 V, for the 5 and 1 Ohm resistors, respectively. The power dissipated across each resistor (Joule heat) is $P = \Delta V \times I$, hence 500 and 100 Watts. (Units: The units of Volts are Joule/Coulomb and Amps are Coulombs/sec)

2. In the above problem, let us put the two resistors in parallel. Calculate now the voltage drop and Joule heating produced in each resistor.

   Solution:

   For resistors in parallel the voltage drop is the same across both, and equal to the battery voltage, i.e. 60 V. To calculate the power, first find the current through each resistor. The total current is

   $$I = V \times (\frac{1}{R_1} + \frac{1}{R_2}) = 72\ Amps$$

   Ohms law also says that $V = I_1 R_1 = I_2 R_2$, and using the fact that $I = I_1 + I_2$, solve for I_1:

   $$I_1 = I_2 \frac{R_2}{R_1} = (I - I_1)\frac{R_2}{R_1}$$

   After a little algebra:

   $$I_1 = I \frac{R_2}{R_1 + R_2} = 72 \times \frac{1}{5+1} = 12\ Amps$$

   Then $I_2 = 60\ Amps$. So the power is 720 and 3600 Watts, respectively.

3. The power station problem considered in class produces power of 1000 MW that is transmitted across a line with resistance 2.2 Ohms at an initial voltage of 800,000 Volts. Find the voltage at the user end. What is the user end voltage when the initial voltage is halved?

   Solution:

$$I = \frac{P}{V} = \frac{1000\ MW}{800,000\ V} = 1250\ Amps$$
$$V = IR = 1250\ Amps \times 2.2\ Ohms = 2750\ Volts$$

   lost in the transmission line, so the final voltage is $800,000 - 2750 = 797,250$ V.
   If initial voltage is 400,000 V, then

$$I = \frac{P}{V} = \frac{1000\ MW}{400,000\ V} = 2500\ Amps$$

$$V = IR = 2500\ Amps \times 2.2\ Ohms = 5500\ Volts$$

   is lost, so final is 394,500 V.

4. A bulb has a power rating of 60 Watts and is connected to the outlet voltage of 115 volts. Calculate the resistance in Ohms, and find the charge passing through it in 5 minutes in Coulombs. (For this calculation, the current may be taken as DC)

   Solution:

   Since the power is 60 Watts, and the voltage is 115 volts, we use the equation $P = IV$ relating power to the current and voltage, to deduce that the current is $60/115 = 0.52$ Amps. Therefore the resistance, from Ohm's law is $R = V/I = 115/.52 = 220.4$ Ohms.

   The charge flowing through the resistance is given by multiplying the current I with the time t as $Q = I\ t = 0.52 \times 300 = 156$ Coulombs.

5. A hot-tub heater with resistance of 11.5 Ohms is used in a household with voltage 115 volts, for 2 hour every morning. Assuming that it is used to heat up water at 80% efficiency, and that the temperature boost required is 50^0C, what is the quantity of water used each day? What are the electricity charges for this usage per month? (Assume 12 cents/ kWH charges).

   { This problem requires you to bring together two concepts learned in the lectures at different times, and will help you to develop a global view. Break it up into two parts, one using Ohm's law and the other using heat capacity and then connect them}

Solution:

As suggested we break the problem into two parts. In the first part we ask how many Joules are consumed by the heater. The second part calculates the energy required to heat up m kG of water by 50^0 Celsius as a function of m using the heat capacity of water. Equating the results of the two parts, we obtain the mass of water. Finally we compute the electric charges by multiplying the result of the first part by the rate, remembering that the efficiency factor does not matter to the power company, we pay for what we consume, whether we use it well or not is our problem, not that of the company!

First part: Since the resistance is given, we can calculate the power consumed by using the formula relating power to the voltage and the resistance: $P = V^2/R = 1150$ W. We write this as 1.15 kW. In two hours, we use an energy $2\ hours\ \times 3600\ (seconds/hours) \times 1.150\ 10^3\ W$ $= 8.3 \times 10^6$ J.This is used at 80 % efficiency to heat up the water, so the amount of heat actually absorbed by the water is $E_{used} = .8 \times 8.3 \times 10^6 = 6.64 \times 10^6$ J.

Second part: If we take the mass of water to be m kG, then we can calculate the total amount of heat required to boost up the water temperature by 50^0 Celsius as $E_{used} = mass \times heat\ capacity \times temperature\ difference = m\ (kg) \times 4.2\ (kJ)/(kG^0C)\ 50^0C = m \times 210\ kJ$.

We can then equate the two expressions for the energy used

$E_{used} = 6.64 \times 10^6\ J = m \times 2.10 \times 10^5\ J$, with the solution

$$m = \frac{6.64 \times 10^6}{2.10 \times 10^5} = 3.16 \times 10^2 = 31.6\ kG.$$

This is equal to 31.6 Litres i.e. 8.36 Gallons (using 1 Gallon = 3.78 Litre).

The cost of the usage is found from the energy drawn by the hot tub rating that we calculated above 1.15 kW. Thus in a day we use 2.3kWH and hence in a month 69 kWH. At the given charges this will cost us $69 \times .12 = 8.3$ \$ per month.