Elementary Physics of Energy

Homework 6 Solutions

1. Assumihng an insolation of $\frac{1000\ Btu}{ft^2}$ and an efficiency of 20% means that the actual energy collected is $\frac{200\ Btu}{ft^2}$. The total required energy for heating is $2 \times 5 \times 10^7\ Btu$, or $10^8\ Btu$. Spread over the heating season of 180 days, this requires a daily input of $\frac{10^8\ Btu}{180} = 5.56 \times 10^5\ Btu$. Divide this by the energy collected to get the required collector area:

$$\frac{5.56 \times 10^5\ Btu}{\frac{200\ Btu}{ft^2}} = 2778\ ft^2$$

2. This calculation is similar to Example 4.1 in the text. Find the amount of energy required to heat the water using the formula $\Delta Q = mc\Delta T$, i. e.

$$\frac{1500\ gal}{day} \times \frac{8\ lbs}{gal} \times \frac{1\ Btu}{lb \cdot^0 F} \times 50^0 F = 6 \times 10^5\ Btu$$

where the specific heat was chosen in convenient units. Since the collector is only 25% efficient it is only able to capture 275 Btu/ft^2, so the necessary area is $\frac{6 \times 10^5\ Btu}{275\ Btu/ft^2} = 2182\ ft^2$.

3. From page 103, concrete can store $\frac{22\ Btu}{ft^3 \cdot^0 F} = c_V$. This is exactly like a specific heat except exchanging mass for volume (which are related via density). Hence we can write $\Delta Q = V \times c_V \times \Delta T$ then solve for V, the volume.

$$V = \frac{\Delta Q}{c_V \times \Delta T} = \frac{2 \times 10^5\ Btu}{\frac{22\ Btu}{ft^3 \cdot^0 F} \times 30^0 F} = 303\ ft^3$$

Divide the volume by the area to get the thickness:

$$\frac{303\ ft^3}{1000\ ft^2} = 0.3\ ft$$

4. From the graphs in lecture 12 one can deduce that the peak in the solar spectrum is in the visible part of the electromagetic spectrum, or approximately 400 - 700 nanometers. The formula to convert to frequency and energy are also in this lecture:

$$E = \frac{hc}{\lambda} = h\nu$$

So the frequency ranges from $\nu = c/\lambda = \frac{3\times 10^8\ m/s}{400\times 10^{-9}\ m} = 7.5 \times 10^{14}\ Hz$ to $\nu = \frac{3\times 10^8\ m/s}{700\times 10^{-9}\ m} = 4.3 \times 10^{14}\ Hz$. Similarly, the energy range is from $E = h\nu = (6.63 \times 10^{-34}\ J \cdot s)(7.5 \times 10^{14}/s) = 5 \times 10^{-19}\ J$ to $2.85 \times 10^{-19}\ J$.

5. When an atomic transistion emits a photon the relationship between the energy difference and wavelength is $\Delta E = \frac{hc}{\lambda}$, so

$$\Delta E = \frac{(6.63 \times 10^{-34}\ J \cdot s)(3 \times 10^8\ m/s)}{589 \times 10^{-9}\ m} = 3.4 \times 10^{-19}\ J$$

This transition is interesting for a couple of reasons. It is used in the common orange colored street lamps, and has also become a tool of modern astronomy, where high powered laser beams excite sodium atoms in Earth's outer atmosphere to produce artificial "guide stars".

6. Stefan's Law:

$$\frac{P}{A} = \epsilon\sigma T^4$$

So for the Sun, $P/A = 6.4 \times 10^7\ W/m^2$. Now solve the equation for T and use $P/A = 3.2 \times 10^7\ W/m^2$:

$$T = (\frac{P}{A} \times \frac{1}{\epsilon\sigma})^{1/4} = 4.9 \times 10^3 K$$