Physics 2

Elementary Physics of Energy

Practice Final Solutions

1. Two resistors with resistances 2 Ohms and 3 Ohms are connected to a 10 V battery,
(a) in series
(b) in parallel.
For each case find the voltage drop across each, and the amount of Joule heating produced in each resistor.

Solution:

(a) For resistors in series, the current is the same in both. To find the current in the system use the relation $V_1 - V_3 = V = I(R_1 + R_2)$. Since $V = 10$ Volts and $R_1 + R_2 = 5$ Ohms, then $I = V/(R_1 + R_2) = 2$ Amps. Now the voltage drop across each resistor can be found from $V = I \times R$, so it is 4 V and 6 V, for the 2 and 3 Ohm resistors, respectively. Note that the sum of the voltage dropped across the resistors is equal to the voltage of the battery.

The power dissipated across each resistor (Joule heat) is $P = \Delta V \times I$, hence 8 and 12 Watts. (Units: The units of Volts are Joule/Coulomb and Amps are Coulombs/sec)

(b) For resistors in parallel the voltage drop is the same across both, and equal to the battery voltage, i.e. 10 V.

To calculate the power, first find the current through each resistor. The total current in the system is

$$I = V \times (\frac{1}{R_1} + \frac{1}{R_2}) = 8.3\ Amps.$$

Because $V = I_1 R_1 = I_2 R_2$ and $I = I_1 + I_2$,

$$I_1 = I \frac{R_2}{R_1 + R_2} = 8.3 \times \frac{3}{2+3} = 5\ Amps$$

Then $I_2 = I - I_1 = 3.3\ Amps$. So the power is 50 W for the 2 Ohm resistor and 33 W for the 3 Ohm resistor.

2. A power plant generates 1000 MW which is transmitted by a power line that carries a current of 500 Amps. If the end voltage is 800,000 V what is the resistance of the line?

   Solution:

   First, find the initial voltage:

   $$V = P/I = \frac{1 \times 10^9\ W}{500\ A} = 2 \times 10^6\ V = 2\ MV.$$

   The voltage dropped across the line is $V_i - V_f = 2,000,000 - 800,000 = 1,200,000\ V = 1.2\ MV$. The resistance can then be found from Ohm's Law, $V = IR$, i.e. $R = V/I = (1.2 \times 10^6\ V)/(500\ A) = 2,400\ Ohms$.

3. A hot-tub heater with resistance of 20 Ohms is used in a household with voltage 115 volts, for 2 hour every morning. Assuming that it is used to heat up water at 70% efficiency, and that the temperature boost required is 50^0C, what is the quantity of water used each day? What are the electricity charges for this usage per month? (Assume 25 cents/kWh charges).

   Solution:

   Let the mass of water be m kG. Calculate the total amount of heat energy required to increase the water temperature by 50^0 Celsius using the formula $\Delta Q = mass \times heat\ capacity \times temperature\ difference = m\ (kg) \times 4.2\ (kJ)/(kg^0C) \times 50^0C = m \times 210\ kJ$.

   By inserting Ohm's Law into the equation for Joule heating one arrives at the formula $P = V^2/R = (115\ V)^2/(20\ Ohms) = 661\ W = 661\ J/sec$. So in two hours the energy used $2\ hours \times 3600\ (sec/hour) \times 661\ W = 4.8 \times 10^6$ J. Since the efficiency is 70%, the amount of heat actually absorbed by the water is $E_{used} = .7 \times 4.8 \times 10^6 = 3.3 \times 10^6$ J.

   Then equate the two expressions and solve for the mass:

   $E_{used} = 3.3 \times 10^6\ J = m \times 2.10 \times 10^5\ J$, so

   $$m = \frac{3.3 \times 10^6}{2.10 \times 10^5} = 15.7\ kg.$$

   The cost of usage is found from the energy drawn by the hot tub, i.e. $661\ W = 0.661\ kW$. Thus each day $1.32\ kWh$ are used, so in a month that's $1.32 \times 30 = 39.7\ kWh$. At the given charges this will cost $39.7 \times .25 = 9.92$ \$ per month.

4. A car wash needs 500 gallons of water a day heated from 50^0F to 100^0F. How large a solar collector would be needed to do this? The incident insolation is 1000 Btu/ft^2 and the collector efficiency is 30%.

   Solution:

   Find the amount of energy required to heat the water using the formula $\Delta Q = mc\Delta T$, i. e.

   $$\frac{500\ gal}{day} \times \frac{8\ lbs}{gal} \times \frac{1\ Btu}{lb \cdot^0 F} \times 50^0 F = 2 \times 10^5\ Btu$$

   where the specific heat was chosen in convenient units. Since the collector is 30% efficient it is only able to capture 300 Btu/ft^2, so the necessary area is $\frac{2\times10^5\ Btu}{300\ Btu/ft^2} = 666.7\ ft^2$.

5. A hydel project has a head of 90 meters. Calculate the rate of flow of the lake needed to obtain 1 MW power working at 80% efficiency. If the velocity of water in flow is 20 km per hour, what is the area of the lake? Here the volume flow rate is related to the velocity and the area, by imagining that the lake flows at a steady rate with the surface water discharged into the dam.

   Solution:

   $Energy = mgh$ and $Power = energy/time = mgh/t = (m/t)gh$ where m/t is mass per unit time, hence the flow rate. So

   $$Power = x(kg/sec) \times 9.8(m/sec^2) \times 90\ m.$$

   The actual power required, taking into account the efficiency, is $(1 \times 10^6\ W)/(0.8) = 1.25 \times 10^6\ W$. Put this into the above and solve for the flow rate.

   $$x = \frac{1.25 \times 10^6\ W}{9.8(m/sec^2) \times 90\ m} = 1,417\ kg/sec.$$

   Note that the units of Watts are $kg \cdot m^2/sec^3$. The density of water is 1000 kg/m^3 so the flow rate is 1.42 m^3/sec. To find the area divide this by the velocity, which is $\frac{2\times10^4\ m}{hr} \times \frac{1\ hr}{3600\ sec} = 5.6\ m/sec$, so

   $$\frac{1.42\ m^3/sec}{5.6\ m/sec} = 0.22\ m^2.$$

Problems from Ristinen & Kraushaar:

Ch. 5 Problems (pg. 167): 6, 8
Multiple Choice Questions: 4, 5

Ch. 6 (pg. 207): 4, 18

Additionally you are advised to review the concepts of specific heat and latent heat from the first half of the course.

Physics 2 – Practice Final Solutions – Book Problems

Chapter 5 Problems

6.

$$\frac{2.4\,J}{cm^{3\circ}C} \times (\frac{100\,cm}{m})^3 \times 140\,^\circ C = 336 \times 10^6\,J$$

8.

$$(1 - \frac{5+273}{20+273}) \times \frac{1}{3} = 0.017 = 1.7\%$$

Multiple Choice

4. c. $\frac{10{,}000\,kg}{1s} \times 9.8\frac{m}{s^2} \times 150\,m \times 0.85 = 1.25 \times 10^7\,W$ electric, or $12.5\,MW_e$

5. d. The power in the wind is proportional to the speed cubed. If the speed is tripled from 3 mph to 9 mph, the power is increased by 3^3, or 27 times. The new power is thus $27 \times 5\,kW = 135\,kW$

Chapter 6 Problems

4.

$${}^{238}_{92}\mathrm{U}_{146} \rightarrow {}^{140}_{55}\mathrm{Cs}_{85} + {}^{92}_{37}\mathrm{Rb}_{55} + 6\mathrm{n}$$

18. One plant that makes enough plutonium for 20 bombs generates the following energy in a year:

$$1000\,MW \times 1000\frac{kW}{MW} \times \frac{365.3\,day}{yr} \times \frac{24\,hr}{day} \times 1\,yr = 8.77 \times 10^9 kWh/yr$$

Dividing this result into the total energy generated worldwide gives an effective number of 1000 MW power plants, so:

$$20\,bombs \times \frac{24 \times 10^{11}}{8.77 \times 10^9} = 5,475\,bombs!$$