A conversation[1] regarding

The Extremely Correlated Fermi liquid

(Dec 11, 2020)

Q: Can you explain in brief what you are working on these days?

I have been working together with students, postdocs and collaborators on developing a new theory for describing very strong interactions between electrons- the type that arise in strongly correlated systems of quantum matter. This is called the theory of Extremely Correlated Fermi Liquids, or ECFL in short. The first paper of this theory was written in 2011, and we are continuing to publish refinements and extensions of this theory as well as benchmarking against other reliable techniques. A compilation of reprints can be found in the link: http://physics.ucsc.edu/~sriram/papers/ECFL-Reprint-Collection.pdf

Q: What are strongly correlated systems and what is the type of interaction you mention, and its characteristics?

Strongly correlated systems have been the focus of attention for several years now, these are systems such as cuprate superconductors, heavy Fermion systems and Kondo lattices, where the electrons interact very strongly. This is in contrast to semiconductors and metals such as Al where the interactions between electrons are arguably small enough to treat within perturbation theory, and hence very close to a a Fermi liquid (i.e. a Fermi gas with small corrections described by Landau). The phrase strongly correlated is often used instead of strongly interacting- the latter phrase is more generic and is applied to other systems, such as nuclear matter and hadron physics.

The novel systems are often called Mott Hubbard systems, where one subjects electrons in a single (or few) narrow bands to very strong and local (i.e. short ranged)

[1] A chat with the Physics Alumni Magazine, UCSC Physics Department.

Coulomb interaction between opposite spins. This situation is unlike usual Coulomb interactions in metals, which are long ranged due to the 1/r nature of the electrostatic potential. In Mott Hubbard systems one retains only the very short distance part of the problem, arguing that it is dominant relative to the small bandwidth resulting from weakly overlapping atomic orbitals that form the band. In condensed matter physics one encounters the deceptively simple Hubbard model describing this physics. The Hubbard model has defied theorists despite much effort. The model in 1-dimension is the exception, here it is solvable thanks to highly specialized techniques that are specific to 1-dimension, but the solution does not generalize to higher dimensions.

The special feature of this interaction is that when there is only one electron per atom, the resulting state is an insulator- dubbed the Mott-Hubbard insulator. The resulting insulating state is a major surprise since it flies in the face of a naive expectation that it should be metallic, since the electron band is not completely filled. When one moves away from this density by adding or removing electrons, one ends up with a metal with most unusual characteristics. The ECFL theory is a specialized technique to handle this metallic state.

Q: What are the physical systems described by your theory and why are they of interest?

The prime motivation for developing the ECFL theory is to understand the cuprate superconductors, other systems such as cobaltates are also of interest. As you might know from intense media coverage, the cuprate high Tc superconductors have attracted enormous interest due to their unusual basic physics as well as potential applications in technology. As you might know, the discovery of high temperature superconductivity in cuprate materials in June 1986 by Bednorz and Mueller earned them the Nobel Prize for Physics almost immediately, in 1987. It was recognized very soon afterwards that these materials provide a physical realization of doped Mott-Hubbard systems we mentioned above.

Q: Are there other theories for describing these systems and in what way is your theory different from those?

After 1987 there was an explosion of theoretical interest in these models and systems, with the added ingredient that experimental results were concurrently providing a set of constraints. The data on T dependent resistivity and photoemission played a central role in focussing theories, since these provide a picture of the nature of electronic states. Indeed several ingenious theories have been put forward by the brightest minds in the game. An enormous volume of community effort has gone into developing novel and mostly numerical techniques, such as quantum Monte-Carlo methods and numerical renormalization group methods. However at this stage these are quite far from explaining results of simple experiments. One such experiment is the very broad line shape seen in angle resolved photoemission (ARPES), which probes the energy and wave vector dependence of the electronic spectral function. Another and perhaps most important experiment is the nature of the T dependence of the resistivity, it is unexpectedly almost linear in cuprate systems.

Thus coming up with a quantitative and analytical understanding of the resistivity and photoemission data over the physically interesting range of T has proven to be elusive. This is where the ECFL theory has managed to do very well. The analytical theory of extremely correlated Fermi liquids is a low cost effort carried out primarily at UCSC by my group in the last few years. The results already obtained by us do remarkably well in explaining the resistivity of cuprates in quantitative terms, and also in explaining the highly asymmetric spectral line shapes in photoemission.

The ECFL theory is in fact a program for calculations, rather than a single or few pieces of work- it lays out a methodology where successively better results can be found by working quite hard at a particular expansion. Many theories have proposed a radical departure from Fermi liquid theory. ECFL finds that the result of strong correlations is a ``fragile'' Fermi liquid, having a tiny magnitude of the quasiparticle weight, which in turn is quite sensitive to density and the hopping matrix elements. ECFL also finds an effective Fermi temperature that is orders of magnitude smaller than what observes in weakly interacting systems.

Q: Can you explain what is the fundamental difficulty in treating these strongly correlated models. What is the difference from an ordinary metal such as aluminum? What are Fermi liquids and how do the new systems depart from it?

In Mott-Hubbard systems the main problem differentiating them from standard metals is the extraordinarily large strength of Coulomb interaction, measured relative to the kinetic energy of electrons. This landed the community in a major problem, since no one knew how to handle such strong interactions. Perturbation theory in the sense of

Feynman is the usual recipe for complicated systems, but here one has a model with nothing small to perturb in!

Regarding Fermi liquids, this is a very important type of metal. In fact quantum many-body physics rests on the bedrock of Fermi liquid theory. The Fermi liquid was formulated by Landau and the Soviet school in mid 1950's, who assumed weak coupling, i.e. perturbation theory can be justified. When the coupling is large, we enter the strongly correlated regime, where the premise used to justify this edifice is invalidated.

We know that the resulting state is not a simple one from experiments, since the nature of the resulting state is encoded in the temperature dependent transport and thermal coefficients. Measurements of these objects in the range of T around and below 300K tell us directly that the state is far from a simple Fermi liquid. Unlike simple metals like aluminum, these correlated metals exhibit unusual transport and spectral signatures that are impossible to reconcile with Landau's theory of Fermi-liquids. The almost T-linear temperature dependence of resistivity is a major puzzle, as are the broad shapes of electronic spectral functions.

Q: Can you explain what is the novelty of your theory, and how does it manage to bypass perturbation theory?

My theoretical approach eschews the Feynman-Dyson diagram method, which assumes weak coupling. In its place I adapt and build on the functional differential equation method of Tomonaga and Schwinger. This theory starts in the limit of infinitely strong local interactions, such that electrons are constrained to avoid double occupation in space. This constraint adds an extra term to Fermi's anti-commutation algebra, rendering it non-canonical. Scaling this term with a parameter λ varying between 0 and 1, I connect the non-interacting Fermi gas limit (at $\lambda=0$) with the fully interacting limit (at $\lambda=1$). Within this program the method of Tomonaga and Schwinger is employed to arrive at an exact set of functional differential equations. In the case of quantum electrodynamics or standard many-body problems, analogous functional differential equations are expanded in terms of the coupling-constant, yielding the Feynman-Dyson series. There the two methods were shown by Freeman Dyson to lead to identical results. For the present case of non-canonical Fermions I establish an alternate scheme by showing that one can instead use the interpolation parameter λ to set up a new series. The resulting series has been evaluated to low orders in recent works of my group, leading to very promising results. Results relevant to recent experiments in the physical case of 2-dimensions have also been found from this ECFL methodology for angle resolved photo-emission studies and most recently for the T and density dependent resistivity.

Q: Can you say a bit more about the Feynman-Dyson method and the Tomonaga-Schwinger methods?

In the development of the ECFL theory we learnt about the origin of quantum field-theory and the Nobel winning approaches of Feynman-Tomonaga-Schwinger and Dyson. Historically the strong appeal of visualization ensured that Feynman's diagrammatic methods dominated science as described nicely by Kaiser et. al.*, to the detriment of Tomonaga-Schwinger's methods. In the strong correlation problem we find that the Feynman-Dyson method fails, while the Tomonaga-Schwinger method succeeds.

Q: So you are saying that for very strongly correlated Fermi systems the Feynman method of diagrams fails while the Tomonaga-Schwinger works?

That is essentially correct. For Mott-Hubbard systems treated in ECFL theory, only the Tomonaga-Schwinger method is applicable. The reason is that a Gaussian type clustering property of non-interacting Fermions (the Wicks theorem) is missing for extremely correlated Fermions. Their modified anticommutators make them irrevocably non-Gaussian! The beautiful Feynman diagrams are alas inapplicable, and the Tomonaga-Schwinger method alone survives!

The Tomonaga-Schwinger method is not immediately applicable either. This method yields exact functional differential equations that are non-linear and hence they pose a very difficult problem themselves. In order to solve that problem one had to create a systematic procedure of expanding the exact equations in a well chosen small parameter. A guiding light in this forest of technical difficulties is the Luttinger-Ward theorem. It constrains the Fermi surface of the resulting theory to track the simple Fermi surface of the non-interacting gas. We showed that this constraint survives very strong correlations, if we assume a continuity in some parameter. That same parameter is used in the above expansion. You will see that a considerable body of new methodology had to be developed in order to formulate a systematic and novel procedure for calculations. That procedure is at the heart of the ECFL theory.

Q: What is the qualitative difference between the Mott Hubbard systems and QED treated by Feynman-Tomonaga-Schwinger?

Well QED (quantum electrodynamics) has a complication that is unique to relativistic field theories, namely a divergence of almost all quantities one can calculate, due to the slow falloff of Greens functions at high energies. The procedure or renormalization was invented to deal with that, and one finally deals with renormalized masses etc, which are finite although the bare masses might be divergent. In a non-relativistic theories such as the Mott Hubbard model, there are no infinite terms of this type. Since one has a theory defined on the lattice, there is automatically a lattice cutoff that prevents high energy divergences.

However the Mott-Hubbard system has a major difficulty which is not present in QED. In QED most interesting results are already found from 2nd or 3rd order perturbation theory- after performing a renormalization procedure. However for Mott-Hubbard systems the strength of interaction is so strong that perturbation theory has to be abandoned. New ways of describing interactions through non-canonical operators are required, these are part and parcel of the ECFL theory.

Q: Going back to the historical context, how did Dyson find an equivalence between the works of Feynman-Tomonaga-Schwinger for QED?

Here are a few paras from the entertaining article by Kaiser et. al.*

From *D. Kaiser, K. Ito and K. Hall, *Feynman diagrams in the USA, Japan and the Soviet Union*, Social Studies of Science, **34/6** (December 2004) 879-922

At the Pocono Manor Inn that spring day in 1948, Feynman introduced his diagrams to serve as a bookkeeping device when wading through these complicated calculations. ---As a step along the way, he wanted first to find a reliable way of making perturbative calculations – to write down the algebraic form for these terms without confusing or omitting elements, before worrying about how to coax the infinities into finite numbers. He designed his diagrams to stand in a one-to-one relation with the mathematical terms he aimed to calculate.

Simple as the scheme might have appeared to Feynman himself, however, his listeners at the 1948 Pocono meeting had great difficulty following his energetic presentation. Not only did Feynman suffer frequent interruptions from the likes of Niels Bohr, *Wolfgang Pauli, Paul Dirac, and Edward Teller, he also eschewed formal rules for manipulating his diagrams in favor of more casual rules of thumb, which he hoped to flesh out via worked examples. The interruptions prevented him from doing so, and Feynman managed only to further confuse his listeners. By all indications, Feynman's initial presentation of his diagrams was a flop.*

A few months later, Freeman Dyson, a graduate student at Cornell (where Feynman was teaching and working out his new diagrammatic scheme) supplied what many people had found missing in Feynman's original presentation. After working closely with Feynman throughout the spring of 1948, the two drove cross-country together that summer, on a trip that afforded Dyson the opportunity to do some sightseeing as well as to plumb more deeply into how Feynman's new techniques were meant to work. After their long drive, the two parted company: Feynman stayed in New Mexico for a few weeks to do some work at Los Alamos, while Dyson made his way by bus to Ann Arbor, MI, for the start of the famous summer school on theoretical physics. The main speaker that summer was Julian Schwinger – like Feynman, one of the young guns of US theoretical physics – who was also then working on QED. Schwinger had worked out his own, non-diagrammatic methods to rid QED of its troublesome infinities, at least in the two-photon term – in fact, before Feynman had said a word at the Pocono meeting, Schwinger had delivered a virtuoso, all-day lecture on his new techniques. His arcane mathematical approach likewise occupied his lectures that summer in Ann Arbor. During the summer school session, Dyson managed to talk several times with Schwinger outside of the lecture hall, learning in more detail about the ins and outs of Schwinger's methods. Thus by the middle of the summer of 1948, Dyson – and Dyson alone – had spent intense time working side-by-side with both Feynman and Schwinger, learning informally how each of them went about making calculations in QED.

On the bus-ride back to the east coast after the summer school session, Dyson worked out two key results: first, that all of Feynman's relations between diagram elements and mathematical expressions – Feynman's sometimes vague rules of thumb – could be derived rigorously from the foundations of quantum field theory; and second, that Feynman's and Schwinger's very different-looking approaches were in fact mathematically equivalent.

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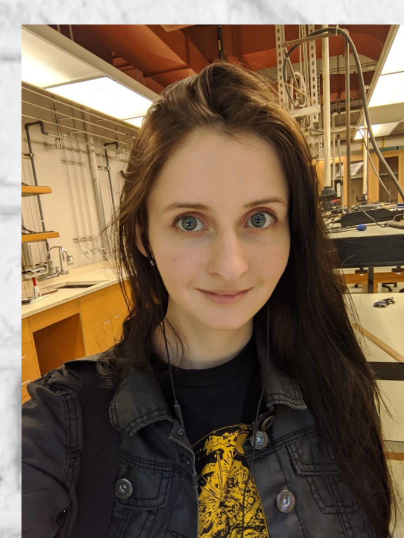

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ECFL Group Pictures