Corrections in : A Sum Rule for Thermal Conductivity and Dynamical Thermal Transport Coefficients in Condensed Matter -I *

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A list of corrections is provided below. A version of the paper with all errors removed is available at
`http://physics.ucsc.edu/~sriram/papers_all/ksumrules_errors_etc/evolving.pdf`.

$$\kappa(k_x,\omega_c) = \frac{-1}{\hbar T k_x \mathcal{L}} \int_{-\infty}^{t} e^{i\omega_c(t-t')}\, dt' \langle [\hat{J}^Q_x(k_x,t), \hat{K}(-k_x,t')]\rangle \quad (7)$$

$$\kappa(\omega_c) = \frac{i}{\hbar\omega_c T}\frac{1}{\mathcal{L}}\left[\langle\Theta^{xx}\rangle + \hbar\sum_{n,m}\frac{p_n-p_m}{\epsilon_n-\epsilon_m+\hbar\omega_c}|\langle n|\hat{J}^Q_x|m\rangle|^2\right]. \quad (13)$$

$$\gamma(\omega_c) = \frac{i}{\hbar\omega_c T\mathcal{L}}\left[<\Phi^{xx}> + \hbar\sum_{n,m}\frac{p_n-p_m}{\epsilon_n-\epsilon_m+\hbar\omega_c}\langle n|\hat{J}_x|m\rangle\langle m|\hat{J}^Q_x|n\rangle\right]. \quad (26)$$

$$\gamma(\omega_c) = \frac{i}{\hbar\omega_c T}D_\gamma + \frac{i\hbar}{T\mathcal{L}}\sum_{n,m}\left(\frac{p_n-p_m}{\epsilon_m-\epsilon_n}\right)\frac{\langle n|\hat{J}_x|m\rangle\langle m|\hat{J}^Q_x|n\rangle}{\epsilon_n-\epsilon_m+\hbar\omega_c}. \quad (27)$$

$$D_\gamma = \frac{1}{\mathcal{L}}\left[<\Phi^{xx}> - \hbar\sum_{n,m}\frac{p_n-p_m}{\epsilon_m-\epsilon_n}\langle n|\hat{J}_x|m\rangle\langle m|\hat{J}^Q_x|n\rangle\right]. \quad (28)$$

$$\gamma(\omega_c) = \frac{i}{\hbar\omega_c T}D_\gamma + \frac{1}{T\mathcal{L}}\int_0^\infty dt e^{i\omega_c t}\int_0^\beta d\tau\langle\hat{J}_x(t-i\tau)\hat{J}^Q_x(0)\rangle, \quad (30)$$

Note added after Eq(42):

For charged systems, a small correction arises from the constraint of zero electrical current (zc) under a thermal gradient. This correction can be included in the present formalism by using Onsager's reciprocity relations, and leads to modified expressions for Eq(18, 40 and 41). These may be written as

$$\int_0^\infty Re\ \kappa_{zc}(\omega)\, d\omega = \frac{\pi}{2\hbar T\mathcal{L}}\left\{\langle\Theta^{xx}\rangle - \frac{\langle\Phi^{xx}\rangle^2}{\langle\tau^{xx}\rangle}\right\}, \quad (18.1)$$

$$\mathbf{L}^* = \frac{\langle\Theta^{xx}\rangle}{T^2\langle\tau^{xx}\rangle} - (S^*)^2, \quad (40.1)$$

$$\mathbf{Z}^*T = \frac{\langle\Phi^{xx}\rangle^2}{\langle\Theta^{xx}\rangle\langle\tau^{xx}\rangle - \langle\Phi^{xx}\rangle^2}. \quad (41.1).$$

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$$J_x^Q = \sum_{\vec{p},\sigma} v_{\vec{p}}^x \left(\varepsilon_{\vec{p}} - \mu\right)\; c_{\vec{p},\sigma}^\dagger c_{\vec{p},\sigma} + \frac{U}{2\mathcal{L}} \sum_{\vec{l},\vec{p},\vec{q},\sigma} \left\{ v_{\vec{l}}^x + v_{\vec{l}+\vec{q}}^x \right\} c_{\vec{l}+\vec{q},\sigma}^\dagger c_{\vec{l},\sigma} c_{\vec{p}-\vec{q},\bar{\sigma}}^\dagger c_{\vec{p},\bar{\sigma}} \quad (61)$$

$$J_x^Q(\vec{k}) = \sum_{\vec{p},\sigma} v_{\vec{p}}^x \left(\varepsilon_{\vec{p}} - \mu\right)\; c_{\vec{p}+\frac{1}{2}\vec{k},\sigma}^\dagger c_{\vec{p}-\frac{1}{2}\vec{k},\sigma} + \frac{1}{2\mathcal{L}} \sum_{\vec{l},\vec{p},\vec{q},\sigma,\sigma'} [U(\vec{q}) \left\{ v_{\vec{l}}^x + v_{\vec{l}+\vec{q}}^x \right\} + \frac{1}{\hbar} \frac{\partial U(\vec{q})}{\partial q_x}\; (\varepsilon_{\vec{l}+\vec{q}} - \varepsilon_{\vec{l}})] c_{\vec{l}+\vec{q}+\frac{1}{2}\vec{k},\sigma}^\dagger c_{\vec{l}-\frac{1}{2}\vec{k},\sigma} c_{\vec{p}-\vec{q},\sigma'}^\dagger c_{\vec{p},\sigma'} \quad (62)$$

$$\begin{aligned}
\Theta^{xx} &= \Theta_E^{xx} - \frac{2\mu}{q_e}\Phi^{xx} - \frac{\mu^2}{q_e^2}\tau^{xx}, \quad \text{with} \\
\chi(\vec{r}_1, \vec{r}_2, \vec{r}_3) &\equiv \frac{-i}{2}\left\{(x_1 - x_3)V_{\vec{r}_2,\vec{r}_3} - (x_2 - x_3)V_{\vec{r}_1,\vec{r}_3}\right\} \text{ and } n_{\vec{r}} = \sum_\sigma c_{\vec{r},\sigma}^\dagger c_{\vec{r},\sigma}, \\
\Theta_E^{xx} &\equiv \frac{1}{2\hbar}\sum t(\vec{\eta})t(\vec{\eta}')t(\vec{\eta}'')(\eta_x + \eta_x')(\eta_x + \eta_x' + \eta_x'')c_{\vec{r}+\vec{\eta}+\vec{\eta}'+\vec{\eta}'',\sigma}^\dagger c_{\vec{r},\sigma} \\
&\quad - \frac{i}{2\hbar}\sum t(\vec{\eta})t(\vec{\eta}')(\eta_x + \eta_x')\chi(\vec{r} + \vec{\eta} + \vec{\eta}', \vec{r}, \vec{r}_j)c_{\vec{r}+\vec{\eta}+\vec{\eta}',\sigma}^\dagger c_{\vec{r},\sigma}\; n_{\vec{r}_j} \\
&\quad - \frac{1}{\hbar}\sum t(\vec{\eta})\chi(\vec{r} + \vec{\eta}, \vec{r}, \vec{r}_j)\chi(\vec{r} + \vec{\eta}, \vec{r}, \vec{r}_l)c_{\vec{r}+\vec{\eta},\sigma}^\dagger c_{\vec{r},\sigma}\; n_{\vec{r}_j} n_{\vec{r}_l} \\
&\quad - \frac{i}{2\hbar}\sum t(\vec{\eta})t(\vec{\eta}')(\eta_x + \eta_x')\left\{\chi(\vec{r} + \vec{\eta} + \vec{\eta}', \vec{r} + \vec{\eta}', \vec{r}_j) + \chi(\vec{r} + \vec{\eta}, \vec{r}, \vec{r}_j)\right\} c_{\vec{r}+\vec{\eta},\sigma}^\dagger c_{\vec{r},\sigma}\;\; c_{\vec{r}+\vec{\eta}+\vec{\eta}',\sigma'}^\dagger c_{\vec{r},\sigma'}. \quad (63)
\end{aligned}$$

$$\begin{aligned}
\hbar\Theta^{xx} &= \mu \sum_{\vec{\eta},\eta',\vec{\sigma},\sigma',\vec{r}} (\eta_x + \eta_x')^2\; t(\vec{\eta})\; t(\vec{\eta'})\; Y_{\sigma',\sigma}(\vec{r} + \vec{\eta})\; c_{\vec{r}+\vec{\eta}+\vec{\eta'},\sigma'}^\dagger c_{\vec{r},\sigma} + \mu^2 \sum_{\vec{\eta},\sigma} \eta_x^2\; t(\vec{\eta})\; \tilde{c}_{\vec{r}+\vec{\eta},\sigma}^\dagger \tilde{c}_{\vec{r},\sigma} \\
&+ \frac{1}{4} \sum_{\vec{\eta},\vec{\eta'},\vec{\eta''},\vec{r},\sigma,\sigma',\sigma''} (\eta_x + \eta_x' + \eta_x'')(2\eta_x + \eta_x' + \eta_x'')\; t(\vec{\eta})\; t(\vec{\eta'})\; t(\vec{\eta''})\; Y_{\sigma'',\sigma'}(\vec{r} + \vec{\eta} + \vec{\eta'})\; Y_{\sigma',\sigma}(\vec{r} + \vec{\eta'})\; \tilde{c}_{\vec{r}+\vec{\eta}+\vec{\eta'}+\vec{\eta''},\sigma''}^\dagger \tilde{c}_{\vec{r},\sigma} \\
&- \frac{1}{4} \sum_{\vec{\eta},\vec{\eta'},\vec{\eta''},\vec{r},\sigma} (\eta_x + \eta_x')(-\eta_x + \eta_x' + \eta_x'')\; t(\vec{\eta})\; t(\vec{\eta'})\; t(\vec{\eta''}) \\
&\left[\left\{\tilde{c}_{\vec{r}+\vec{\eta'},\sigma}^\dagger \tilde{c}_{\vec{r}+\vec{\eta'}+\vec{\eta''},\bar{\sigma}} - \tilde{c}_{\vec{r}+\vec{\eta'}+\vec{\eta''},\sigma}^\dagger \tilde{c}_{\vec{r}+\vec{\eta'},\bar{\sigma}}\right\} \tilde{c}_{\vec{r}+\vec{\eta}+\vec{\eta'},\bar{\sigma}}^\dagger \tilde{c}_{\vec{r},\sigma} - \left\{\tilde{c}_{\vec{r}+\vec{\eta'},\bar{\sigma}}^\dagger \tilde{c}_{\vec{r}+\vec{\eta'}+\vec{\eta''},\bar{\sigma}} - (h.c.)\right\} \tilde{c}_{\vec{r}+\vec{\eta}+\vec{\eta'},\sigma}^\dagger \tilde{c}_{\vec{r},\sigma}\right] \quad (81)
\end{aligned}$$

$$\Delta \sim -\frac{3}{\hbar}\mathcal{L}t^2 \sum_{\sigma,\sigma'} \langle Y_{\sigma',\sigma}(\vec{\eta})\tilde{c}_{\vec{\eta}+\vec{\eta'},\sigma'}^\dagger \tilde{c}_{\vec{0},\sigma}\rangle. \quad (85)$$

$$\Delta = -\frac{3}{2\hbar}\mathcal{L}t^3\beta n(1-n)(2-n) + O(\beta^3). \quad (86)$$

$$\tau^{xx} = -\lim_{k\to 0}\frac{1}{k_x}[\hat{J}_x(k_x), \rho(-k_x)] \quad (A2)$$

$$\sigma(\omega_c) = \frac{i}{\hbar\omega_c}\frac{1}{\mathcal{L}}\left[\langle\tau^{xx}\rangle + \hbar\sum_{n,m}\frac{p_n - p_m}{\epsilon_n - \epsilon_m + \hbar\omega_c}|\langle n|\hat{J}_x|m\rangle|^2\right]. \quad (A5)$$