Excitations in Quantum Spin Systems in Low Dimensional Systems

Sriram Shastry

UC Santa Cruz

Topics:

a) Non linear excitations in magnetic systems:

1) From phonons to solitons in non linear classical lattices

2) Analogous transition from magnons to solitons :
The case of dimerized Heisenberg spins chain.

3) Spinons in 1-d Heisenberg antiferromagnet: Analogy with particle hole excitations in metals

b) Spin one Haldane systems, ground states and excitations, edge states.

c) Two dimensional solvable Heisenberg s=1/2 system, Sr Cu (BO_3)_2 and its triplet excitations, and bi-triplet excitonic excitations

Lattices: Non linear kinds: vs Magnetic systems

Phonons

|

|

|

Magnons

Solitons

*Non Topological

(Non linearity based)

*Topological

|

SPINONS

LATTICES (NON LINEAR)

FERMI PASTA ULAM, HENON HEILES, TODA

$$H = \sum \frac{p_n^2}{2M} + \frac{k}{2}\sum_n (u_{n+1}-u_n)^2 - \gamma \sum (u_{n+1}-u_n)^3 \quad \begin{pmatrix} \text{FPU} \\ \text{HH} \end{pmatrix}$$

or

$$= \sum \frac{p_r^2}{2m} + k \sum_r e^{-(x_{n+1}-x_n)} \quad (\text{TODA})$$

* **PHONONS**

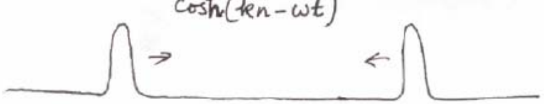
- QUADRATIC P.E. SMALL OSCILLATIONS

$$U_n = u_0 \, e^{\pm i(kn-\omega t)}$$

- u_0 independent of k (velocity)
- SUPERPOSITION GOOD

* **SOLITONS**

$$U_n = \frac{A_k}{\cosh(kn-\omega t)}$$



- A_k depends on k
- STABLE (only long lives)
- NO SUPERPOSITION.
- BUT SCATTER OFF each other

These solitons are built on nonlinearity.

TOPOLOGICAL SOLITONS

* MORE PROTECTION FROM TOPOLOGY – LESS FROM DYNAMICAL CONSERVATION LAWS

* EXAMPLE: POLY ACETYLENE SU-SCHRIEFFER-HEEGER (1980)

ELECTRONS ON AN ELASTIC LATTICE

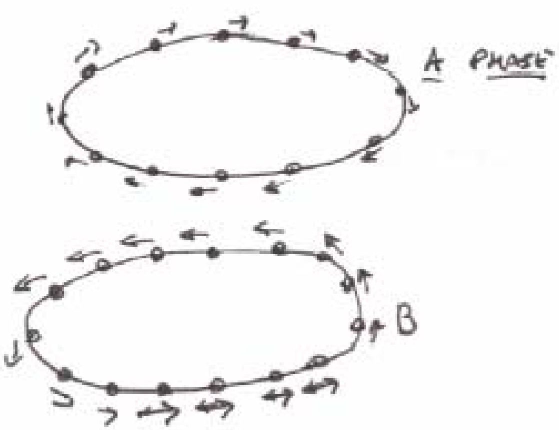
$$H = -t \sum_n \left\{ \left(1 + \alpha (u_{n+1} - u_n)\right) c^{+}_{n+1} c_n + h.c. \right\} + \frac{1}{2M} \sum P_n^2 + \frac{k}{2} \sum (u_{n+1} - u_n)^2$$

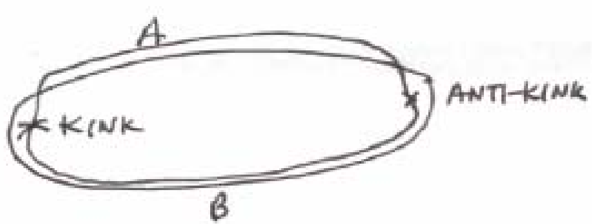
C's are FERMIONS – HALF FILLED

GROUND STATE HAS 2 FOLD DEGENARCY

$u_n = \pm u_0 e^{in\pi}$ $u_0 = \pm|\Delta|$ – PEIERLS DIMERIZATION

$+\Delta \to A$] PHASE
$-\Delta \to B$] PHASE





* KINKS LOCALIZE FRACTIONAL CHARGE (1/2)

* THEY PROPAGATE – LIKE PARTICLES.

Magnons: Isotropic spin interactions

$$H = J \sum S_i . S_j$$

Safeland: 3 dimensions Ferromagnet or Antiferromagnets

Small excitations of the magnetic moments. Live off magnetic Long Ranged Order

Imagine small precession of cones of moments.

Unsafeland : 1 and 2 dimensions, If classical fluctuations don't get you, then quantum fluctuations must!

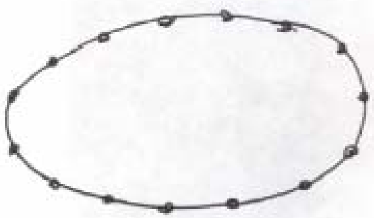
Ashes to ashes, dust to dust,
If Hohenberg don't get you,
Then Heisenberg must!

- T=0 Long Ranged Order at only for d >=2 for any spin
- T>0 Long Ranged Order only for d>=3
- Hence huge range (T small compared to J, d<3) where both quantum and classical fluctuations rule the roost. Many experimental systems, and in response many beautiful theoretical ideas have been proposed.
- 1-d :s=1/2
- 1-d s=1
- 2-d s=1/2

HEISENBERG AFM

HANS BETHE (1932)

$$H = J \sum_{n=1}^{N} \vec{S}_n \cdot \vec{S}_{n+1} \qquad |S_n| = \frac{1}{2}$$



$n + N = n \rightarrow$ PERIODIC BC's

- NO LRO in G.S.
- SPIN LIQUID GROUND STATE

$$\langle S_n^x S_{n+r}^x \rangle = \langle S_n^y S_{n+r}^y \rangle = \langle S_n^z S_{n+r}^z \rangle = \frac{(-1)^r}{|r|} \{\log |r|\}^{1/2}$$

- BETHE'S ANSATZ - CELEBRATED SOLUTION - "QUANTUM SOLITONS".
- INFINITE # CONSERVATION LAWS $I_1, I_2, \ldots I_n$

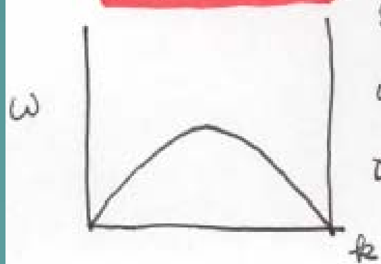
$$I_2 = \sum_n \vec{S}_n \times \vec{S}_{n+1} \cdot \vec{S}_{n+2} \quad \text{(e.g.)} \qquad [I_i, I_j] = 0$$

- Excitation spectrum: (1980) - spinon - Takhtajan & Faddeev.

OLD VIEW: SHARP mode: gapless:

$\omega_k = \pi J |\sin k|$

Des Cloizeaux Pearson



Current view: COMPOSITE 2-PARTICLE SPECTRUM.

~ PARTICLE HOLE SPECTRUM IN 1-d METAL

$$S_k^z \sim \sum C_{k+p}^{+} C_p \gamma_p$$

$C_p \sim$ spinon $\qquad \omega_k = \omega_{k+p}^{sp} + \omega_k^{sp}$

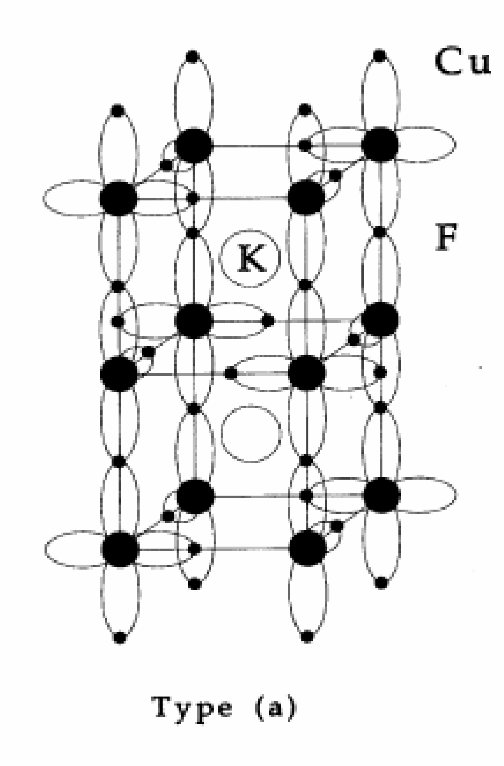
$\sim$ Fermionic. $\qquad \omega_k^{sp} = J\pi/2 \sin k$ / spinon

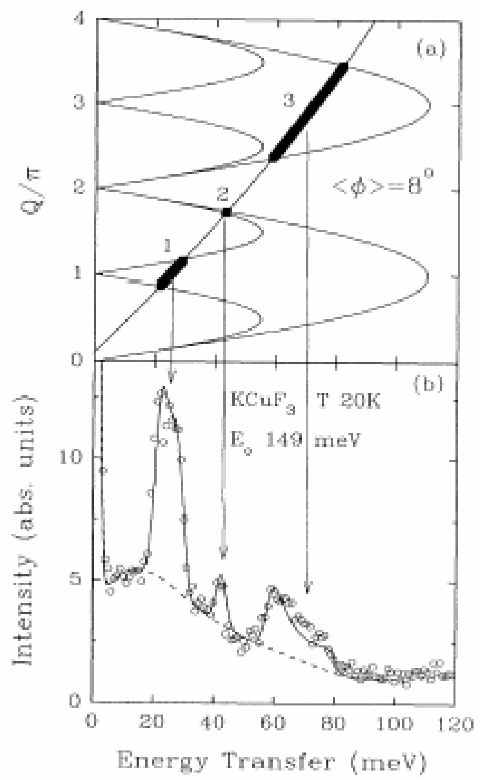
Unbound Spinons in the $S=1/2$ Antiferromagnetic Chain $KCuF_3$

D. A. Tennant, T. G. Perring,[a] and R. A. Cowley
Department of Physics, Clarendon Laboratory, University of Oxford, Parks Road, Oxford, United Kingdom

S. E. Nagler
Department of Physics, University of Florida, Gainesville, Florida 32605
(Received 8 February 1993)

Inelastic neutron scattering has been used to study the temperature dependence of the magnetic response in the one-dimensional $S=1/2$ Heisenberg antiferromagnet $KCuF_3$. The scattering is consistent with that expected for unbound spinon pair excitations.





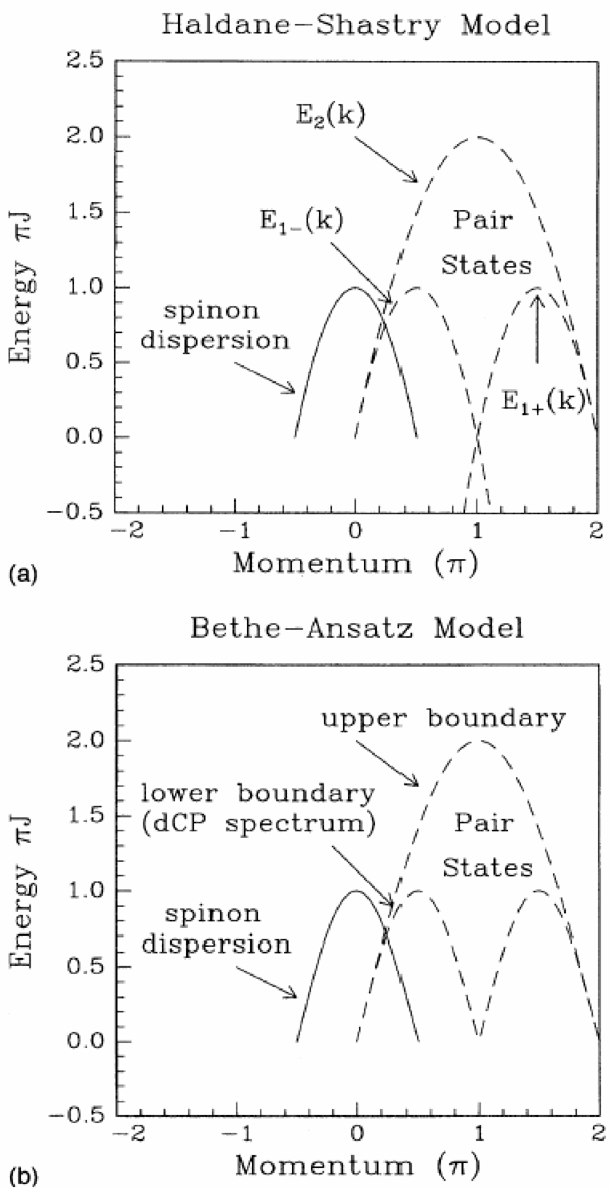
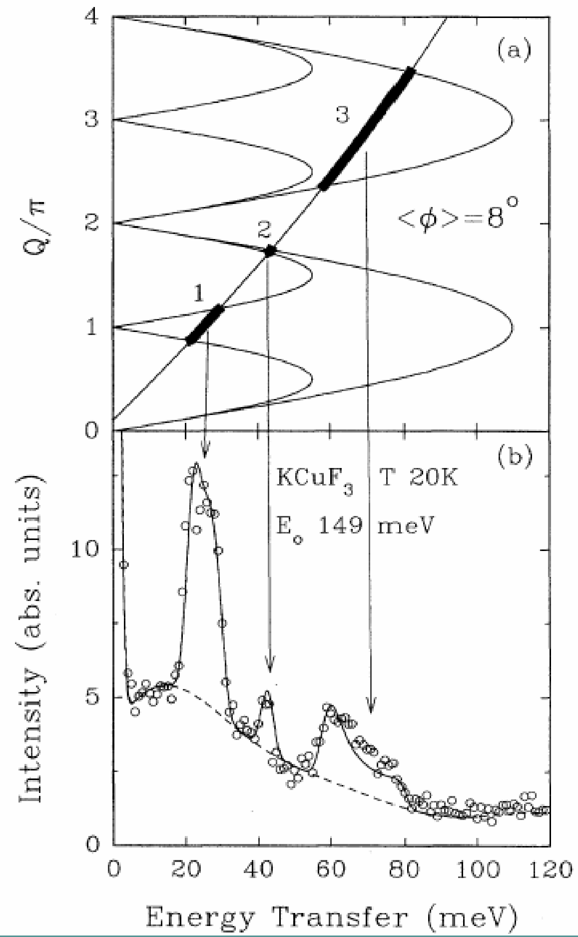


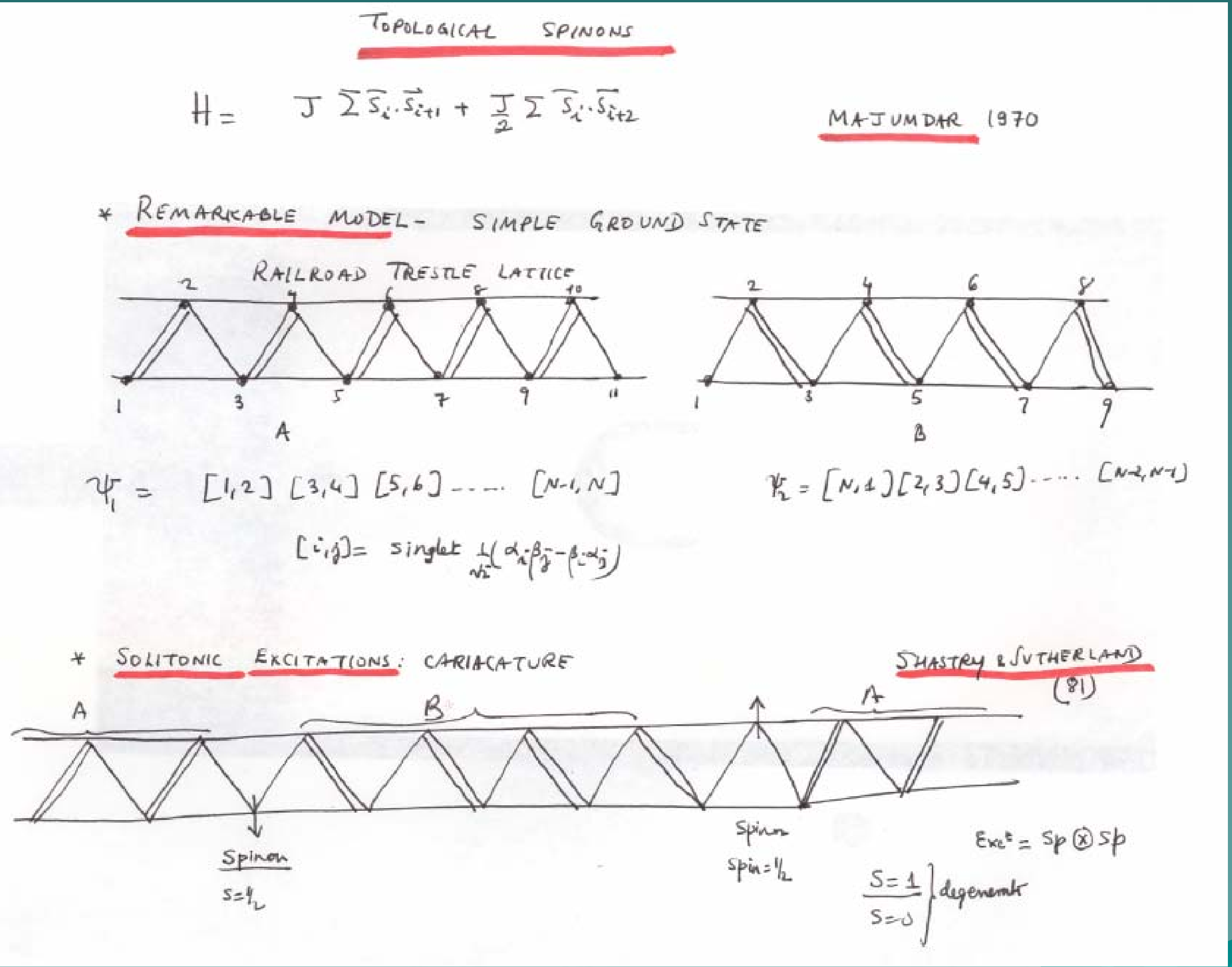
FIG. 1. (a) The spinon dispersion for the Haldane-Shastry model. The two spinon excitations form a continuum. (b) The excitation spectrum for the nearest-neighbor Heisenberg model as proposed by Müller.



VOLUME 47, NUMBER 13 PHYSICAL REVIEW LETTERS 28 SEPTEMBER 1981

Excitation Spectrum of a Dimerized Next-Neighbor Antiferromagnetic Chain

B. Sriram Shastry[a] and Bill Sutherland

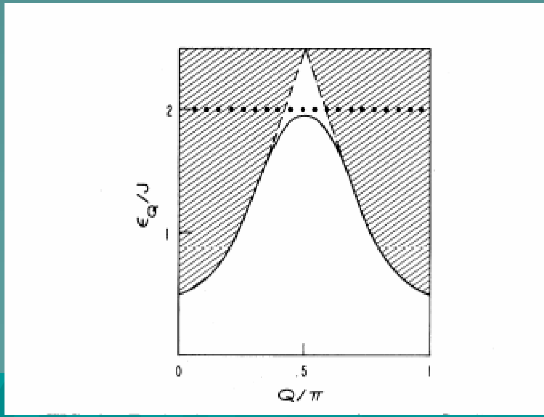


Cu Ge O_3 almost a realization of this J1-J2 chain.

Fractionalized gapped spin excitations

4fold degeneracy

S=1, S=0



Spin 1 Haldane chain:

Heisenberg model with S=1

Haldane showed it has a gap in the excitation spectrum, no LRO. This is another kind of spin liquid- a gapped spin liquid aalogous to the Quantum Hall state of Laughlin. Topological term in action.

$W(k) = J (\text{Cos}[k] + 1 + \Delta)$, where $\Delta > 0$

PHYSICAL REVIEW B VOLUME 54, NUMBER 10 1 SEPTEMBER 1996-II

Y_2BaNiO_5: A nearly ideal realization of the $S=1$ Heisenberg chain with antiferromagnetic interactions

Guangyong Xu
Department of Physics and Astronomy, The Johns Hopkins University, Baltimore, Maryland 21218

J. F. DiTusa
Department of Physics and Astronomy, Louisiana State University, Baton Rouge, Louisiana 70803

T. Ito and K. Oka
Electrotechnical Laboratory, Tsukuba 305, Japan

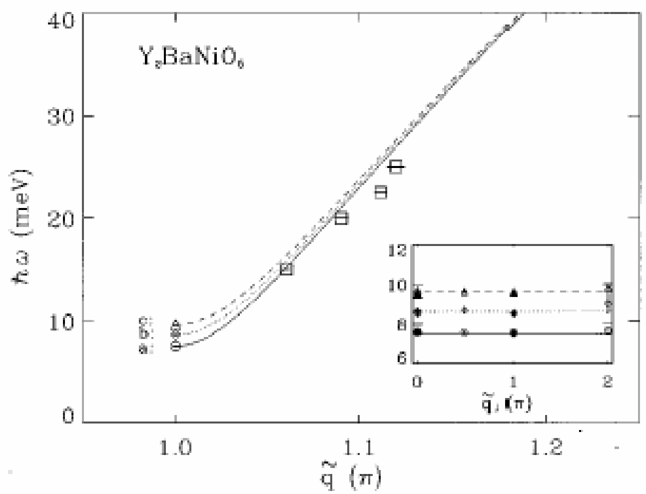
H. Takagi
The Institute for Solid State Physics, University of Tokyo, Roppongi, Tokyo 106, Japan

C. Broholm
Department of Physics and Astronomy, The Johns Hopkins University, Baltimore, Maryland 21218
and National Institute of Standards and Technology, Gaithersburg, Maryland 20899

G. Aeppli
NEC Research Institute, 4 Independence Way, Princeton, New Jersey 08540
(Received 4 June 1996)

$$S^{\alpha\alpha}(\tilde{q},\omega)=\frac{2}{3}(-\langle\mathcal{H}\rangle/L)\frac{1-\cos\tilde{q}}{\hbar\omega_{\alpha}(\tilde{q})}\delta[\hbar\omega-\hbar\omega_{\alpha}(\tilde{q})]. \quad (2)$$

$$\hbar\omega_{\alpha}(\tilde{q})=\sqrt{\Delta_{\alpha}^{2}+v^{2}\sin^{2}\tilde{q}+A\cos^{2}\frac{\tilde{q}}{2}}.$$



ESR gap is at k=0

Neutrons measure gap at all k, smallest gap at k= pi

One Finds

ESR gap = 2 (gap @k= Pi)

Understanding is in terms of two spin 1particles combining to give spin 2,1,0 and if there is no bound state then above formula is true.

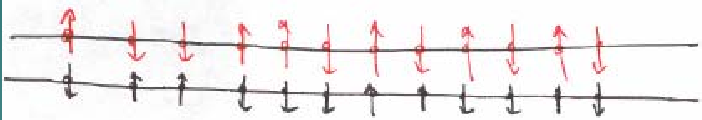
Edge states and understanding the Haldane state via AKLT model.

Affleck Kennedy Lieb Tasaki : AKLT

$S=1$ model

$S=1$ = symmetrization of two $s=\frac{1}{2}$ objects

: Take two spin chains & "fuse" them!



$(\uparrow \otimes \downarrow)$ Symmetrize to get $S=1$

$S_z = S_z + S_z$

e.g.:

$\alpha\,\alpha \rightarrow 1$

$\alpha\,\beta \rightarrow 0$

$\beta\,\alpha \rightarrow 0$

$\beta\,\beta \rightarrow -1$

→ Gapped state with

$\langle S^z_0 S^z_r \rangle \sim e^{-r/\xi}$ $\xi \cong 6$

G.S. for $H = \sum\left(\vec{S}_i \cdot \vec{S}_{i+1} + \frac{1}{3}\left(\vec{S}_i \cdot \vec{S}_{i+1}\right)^2\right)$ { Projector on cross bond singlet!

Edge states

Edge states seen in ESR of truncated $S=1$ chains! NENP

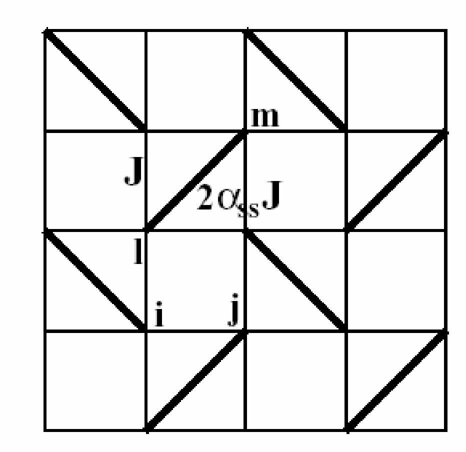
Affleck. Halperin (90)

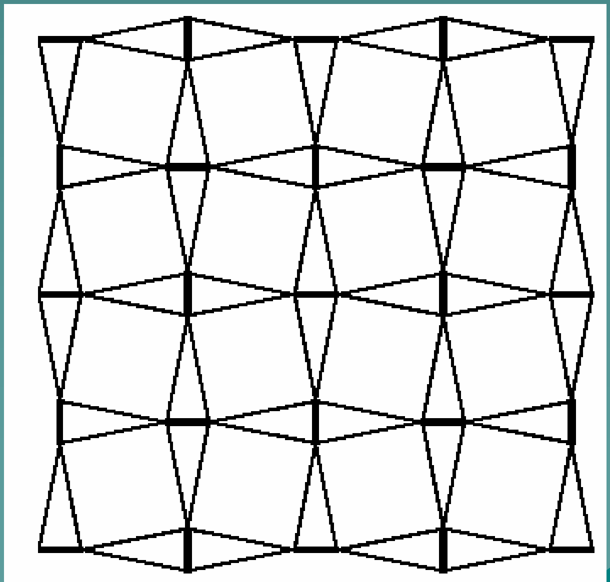
Two dimensions and Sr Cu_2 (BO_3)_2: Realizing a solvable 2d spin ½ model. (only QS model in 2-d)

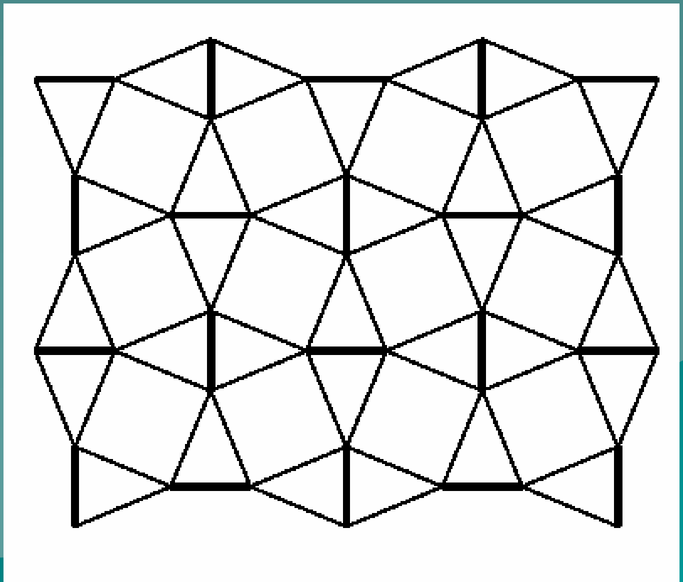
Remarkable trick of nature:toplogical equivalence bewteen

Lattice of Shastry-Sutherland (1981) and Sr Cu_2 (BO_3)_2

$$H = J\Sigma_t H_t = J\left(\sum_{\langle i,j\rangle}\vec{S}_i.\vec{S}_j + 2\alpha_{SS}\sum_{\langle l,m\rangle}\vec{S}_l.\vec{S}_m\right)$$



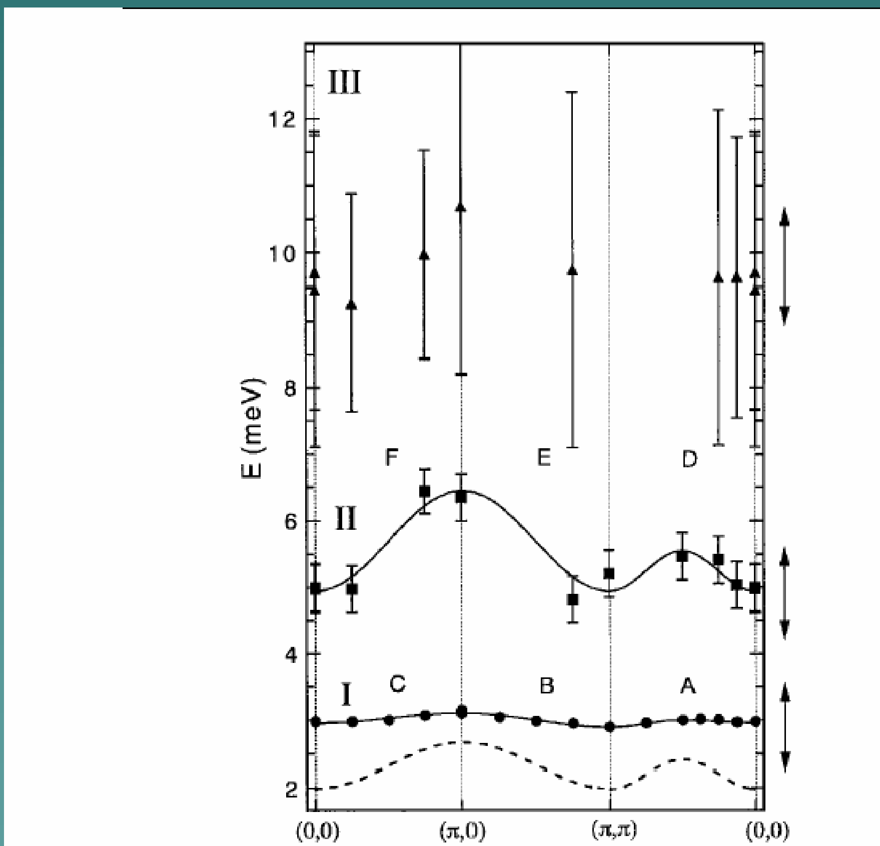




VOLUME 84, NUMBER 25 PHYSICAL REVIEW LETTERS 19 JUNE 2000

Direct Evidence for the Localized Single-Triplet Excitations and the Dispersive Multitriplet Excitations in $SrCu_2(BO_3)_2$

H. Kageyama,[1,*] M. Nishi,[2] N. Aso,[2] K. Onizuka,[1] T. Yosihama,[2] K. Nukui,[2] K. Kodama,[3] K. Kakurai,[2] and Y. Ueda[1]



Single exciton is localized by lattice frustration.

Bi Exciton is mobile..

Magnetization plateaus: analogy with QHE problem:

Excitations are still exciting we have gone past magnons

Beyond magnons, there are a few unifying themes:

Excitations are signatures of parent states: variety is huge

- Toplogical solitons (degenerate gs’s)
- Solitons (xy type models)
- Solitons: constituents of particle hole type two parameter excitations
- Excitons (broken singlets)
- Multi excitons