A Model For All Seasons:

Magnetism and Superconductivity in 2-d

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UC Davis Colloquium

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The exactly solved model invented by Sutherland and the speaker in 1981 has very recently lead to a class of two dimensional systems SrCu_2(BO_3)_2, Tm B_4.... that are currently exciting.

In this talk I will summarize the recent experimental systems that have been found, and also some key ideas from quantum antiferromagnetism and Mott Hubbard physics that are tested.

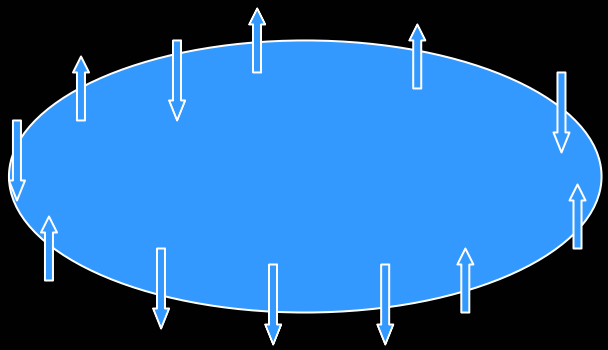
The underlying lattice
has apparently an ancient history dating back to the concerns of Archimedes, and according to many workers, the lattice and the model have an intriguing future as well, providing a testing ground for current ideas on unusual quantum phenomena.

- The Heisenberg type Models on the SSL:
- Early RVB (1969-1981) (Majumdar(1969), Anderson- Fazekas (1973), S-Sutherland(1981))
- Prehistory 200BC Archimedes and his tilings.
- Mott Hubbard Physics: In response to High Tc: The doped spin liquid as the big hope.---what is a spin liquid?
- $SrCu_2(BO_3)_2$ Kageyama and the Magnetization Plateau Story
- More SSL Compounds: R2T2M class Rare earth tetraborides.
- Doping the spin liquid- Shastry -Kumar (2001) and Schmalin-Trivedi (2007)

Early story of RVB

1969 Chanchal K Majumdar, Tata Institute Bombay

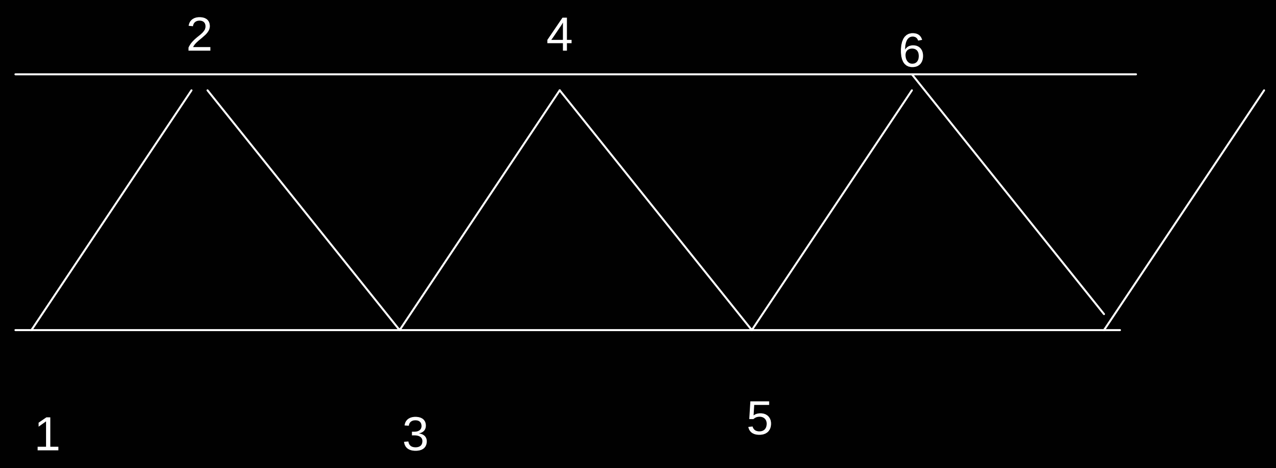
P. W. **Anderson**, **Mater**. **Res**. **Bull**. 8, 153 (1973)



$$\mathrm{H}= \sum_j (\vec{S}_i.\vec{S}_{i+1}) + \alpha \sum_j (\vec{S}_i.\vec{S}_{i+2})$$

$$\alpha = 1/2$$

Exact solution: only solution other than Bethe’s 1932 solution!!



$$\psi_1 = [1,2][3,4][5,6].[N-1,N] \qquad \psi_2 = [2,3][4,5][6,7][N,1]$$

$$\psi_A = \psi_1 - \psi_2 \quad \psi_B = \psi_1 + \psi_2 \qquad \langle \psi_B | \psi_A \rangle \sim O(\exp -N)$$

$N/2 even(odd) \psi_A(\psi_B)$ is the ground state and the other is a low lying excitation with $\delta E \sim O(\exp -N)$

Thermodynamic limit gives a 2 fold degeneragte gs, with solitonic excitations a la Polyacetylene

Shastry Sutherland 1981 (Phys Rev Letts)---discovery of spinons, excitations with spin half rather than spin one as in broken symmetric cases

Affleck Lieb 1988 elevated this to a “theorem”: either broken symmetry (parity) or gapless excitations for spin half particles.

Anderson (1973) Cambridge preprint!!

Revived Pauling’s idea on resonance and gave a rather poor energy gs for Heisenberg antiferromagnet on the triangular lattice:

He did not fool us with the poor numbers emerging from his calculation: We were thoroughly impressed in 1973 by the originality and depth of his ideas!!

However, we were in a minority since there are only 5 or 6 other papers between 1973 and 1986 citing Anderson RVB, and 3 of them are mine and 2 by Anderson!!

The S Sutherland Lattice and the spin model

- 1981 at Slat Lake city withDan Mattis and Bill Sutherland: After a riveting talk by Bob Schrieffer on fractional charge in the polyacetylene model, we chatted with Bill saying that polyacetylene is the same as the Majumdar model!!
- Spinon paper (1981) PRL particles with spin half emerge as fractionalized objects, instead of spin 1 excitations such as magnons.
- How about higher dimensions? Triangles hold the key and frustration was in the air (Les Houches Ill Condensed Matter 1978 with Anderson/Toulouse)

- The SS model on the “funny lattice”

EXACT GROUND STATE OF A QUANTUM MECHANICAL ANTIFERROMAGNET

B. Sriram Shastry and Bill Sutherland

$$H = \sum_{\langle ij\rangle} h_{ij} + 2\alpha \sum_{\langle \ell m\rangle} h_{\ell m} - h \sum_i S_i^z$$

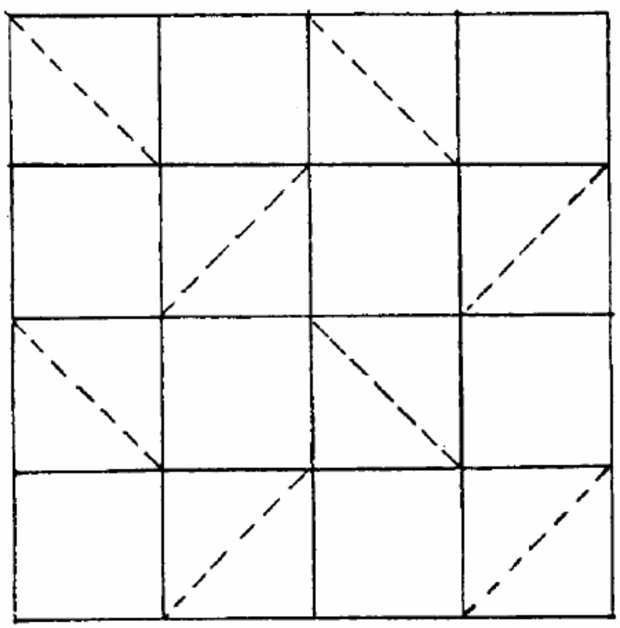


Fig. 1. The Lattice

A. QUANTUM CASE Our solution is inspired by the surprising solution of Majumdar's, [6] for a linear chain Heisenberg antiferromagnet with a nearest and next nearest neighbor coupling which is half as strong as the first. We first observe that the state

$$|\psi\rangle = \prod_{\langle \ell m\rangle} [\ell,m] \qquad (3)$$

. We further show that E_0

saturates a lower bound to the ground state energy for suitable values of the parameters and hence $|\psi\rangle$ is the rigorous ground state.

The nature of ground state correlations in $|\psi\rangle$ is seen to be liquid like, with only short ranged order and hence we have termed it the "quantum spin liquid" phase (Q.S.L.). This wave function is of the type suggested by Anderson [1] and is familiar from valence bond theory in chemistry. For α smaller than the bounding

Physica 108B (1981) 1069-1070
North-Holland Publishing Company

B. ISING LIMIT We have studied the ground state degeneracy of the Hamiltonian in the Ising limit $J_x = J_y = 0$ and $h = 0$ for $s = 1/2$. It is readily established that for $\alpha < 1$, the ground state entropy is $O(1)$ whereas for $\alpha > 1$, it is of $O(N)$. The case $\alpha > 1$ is quite simple

C. CLASSICAL LIMIT As $s \to \infty$, the singlet state is the ground state only as $\alpha \to \infty$. However, for arbitrary α, we have succeeded in determining the ground state exactly in the isotropic limit. We observe that the classical planar (i.e., x-y) and Heisenberg model share a ground state for the triangular Hamiltonian (5). Each triangle has a two-fold discrete "chiral" degeneracy, over and above the continuous degeneracy, of the sort discovered by Villain in similar systems [8]. The optimum twist

Finally we would like to point out the remarkable property of the ground state $|\psi\rangle$ for $s = 1/2$ in the case $\alpha > 1$ and $J_x \neq J_y \neq J_z$. In this case the ground state possesses rotation invariance (being a singlet) even though the Hamiltonian does not. This is the only example of symmetry breaking in reverse that we are aware of. This provides an exmple in non-relativistic physics where the invariance of of the vacuum exceeds the invariance of the world [9].

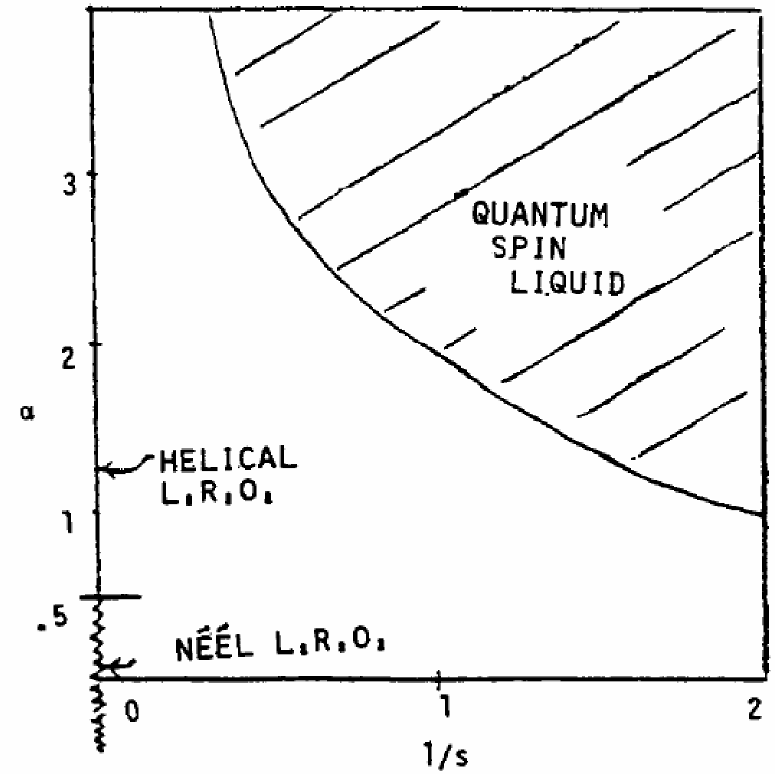


Fig. 2. The Phase Diagram

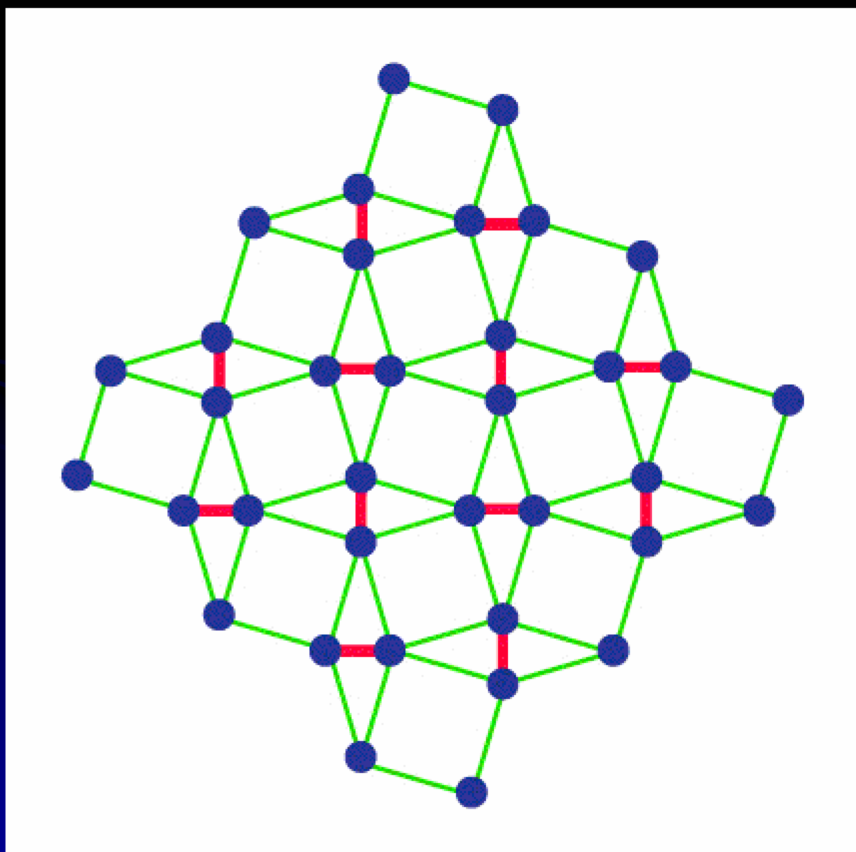
VOLUME 82, NUMBER 15 PHYSICAL REVIEW LETTERS 12 APRIL 1999

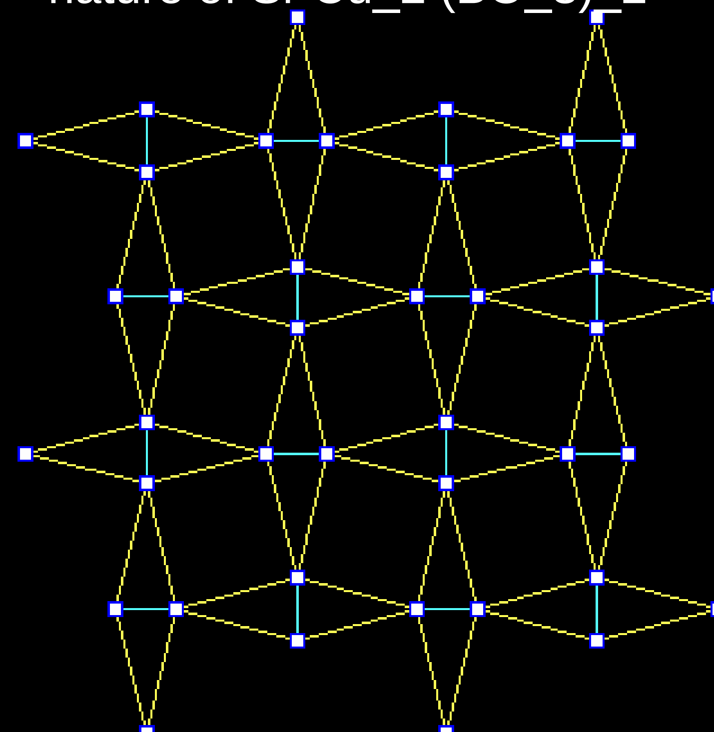
Exact Dimer Ground State and Quantized Magnetization Plateaus in the Two-Dimensional Spin System $SrCu_2(BO_3)_2$

H. Kageyama,[1,2,*] K. Yoshimura,[1,3,†] R. Stern,[3] N. V. Mushnikov,[2] K. Onizuka,[2] M. Kato,[1] K. Kosuge,[1] C. P. Slichter,[3] T. Goto,[2] and Y. Ueda[2]

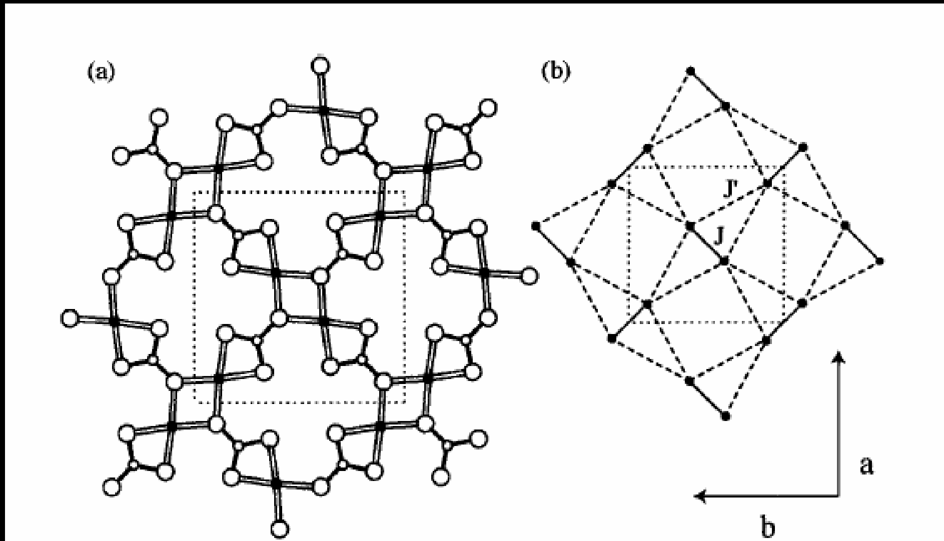
1995 Kageyama et al

Discovery of spin gapped nature of Sr Cu_2 (BO_3)_2





$SrCu_2(BO_3)_2$



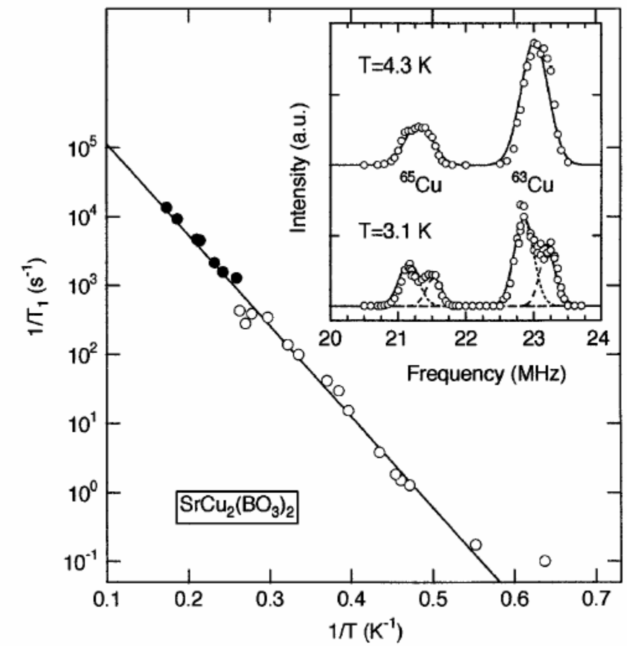
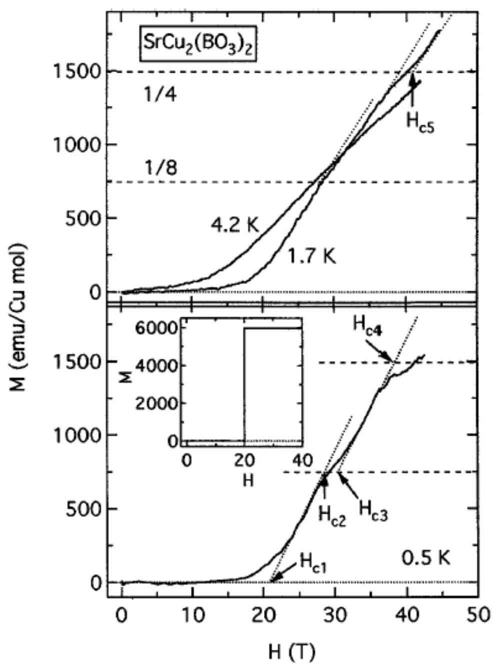
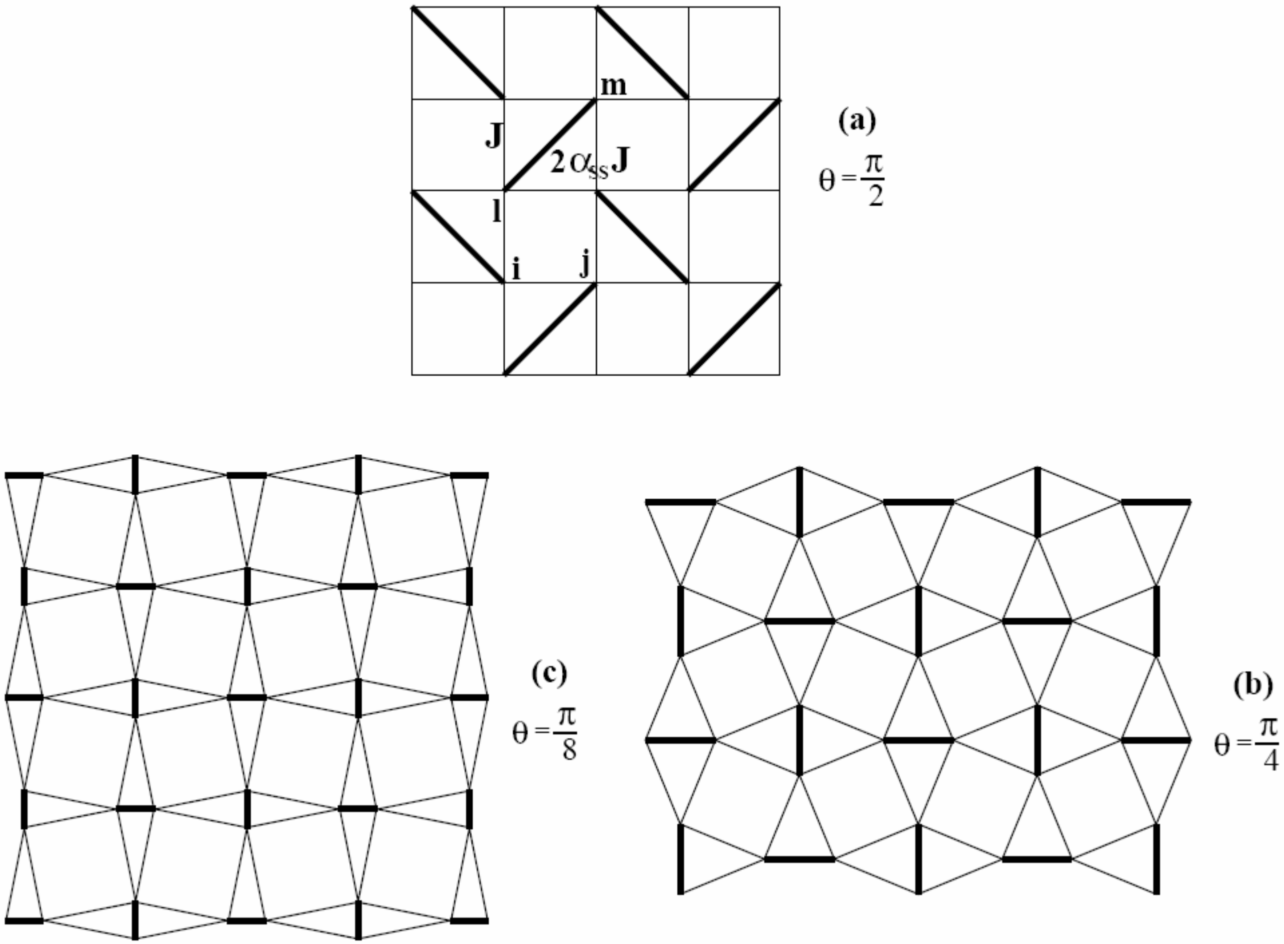


FIG. 3. Temperature dependence of $1/T_1$ measured by saturation recovery method at Kyoto University (open) and by inversion recovery method at University of Illinois at Urbana-Champaign (closed). The solid line shows the activated T



Theoretically, the 2D spin network of $SrCu_2(BO_3)_2$ is equivalent to that for which B. S. Shastry and B. Sutherland solved the ground state exactly [Physica (Amsterdam) **108B**, 1069 (1981)].



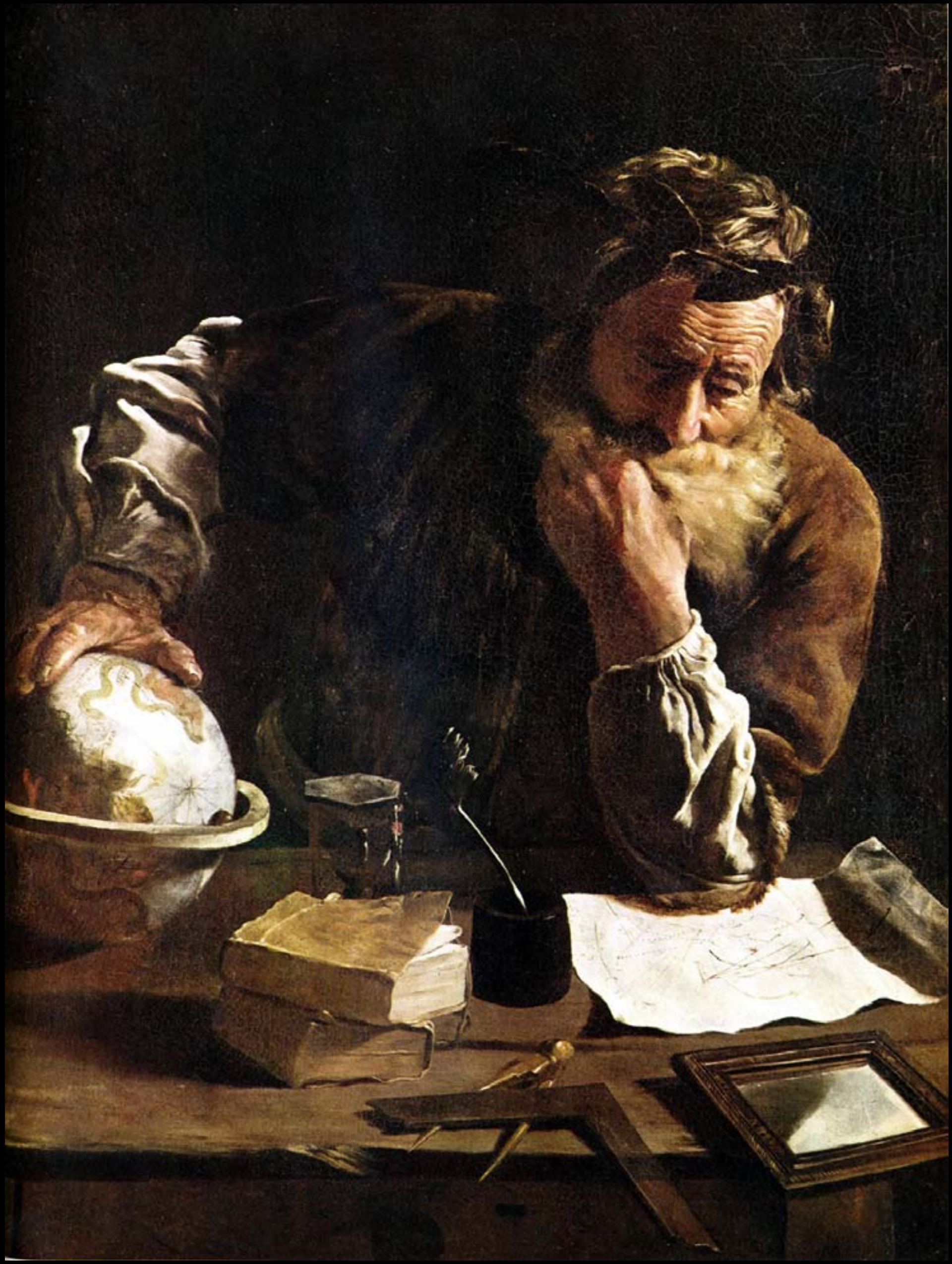
J_{diag} > 1.4 J is now physically reasonable since distances are so arranged. J_{diag} ~ 60K and J~45K experimentally

But the lattice was known to at least two people in history, Archimedes and J Kepler!!

Archimedes was born *c.* 287 BC in the seaport city of Syracuse

Archimedes is considered to be one of the greatest mathematicians of all time. (Wiki)

He thought about tiling the 2-D plane with symmetric polygons:

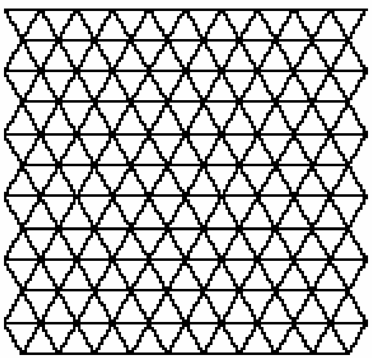


Quantum magnetism in two dimensions: From semi-classical Néel order to magnetic disorder

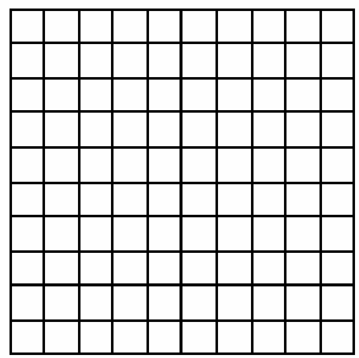
J. Richter[1], J. Schulenburg[2], and A. Honecker[3]

Archimedean Lattices

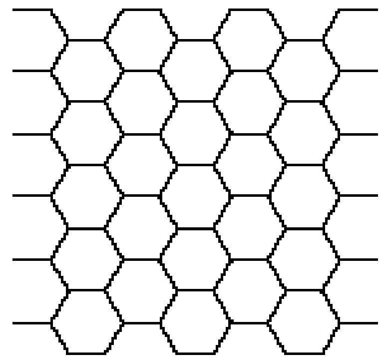
Johannes Kepler (*Harmonice Mundi*, 1619)



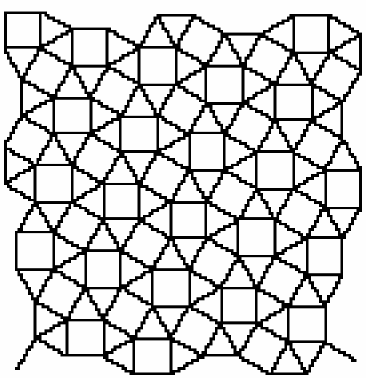
T1: 3^6 = triangular



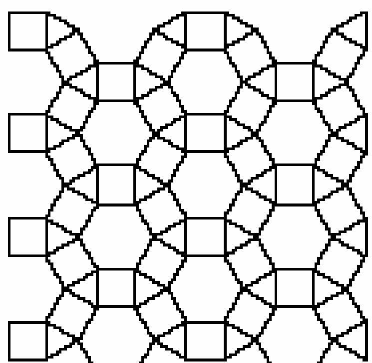
T2: 4^4 = square



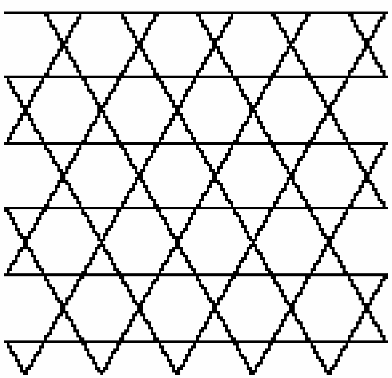
T3: 6^3 = honeycomb



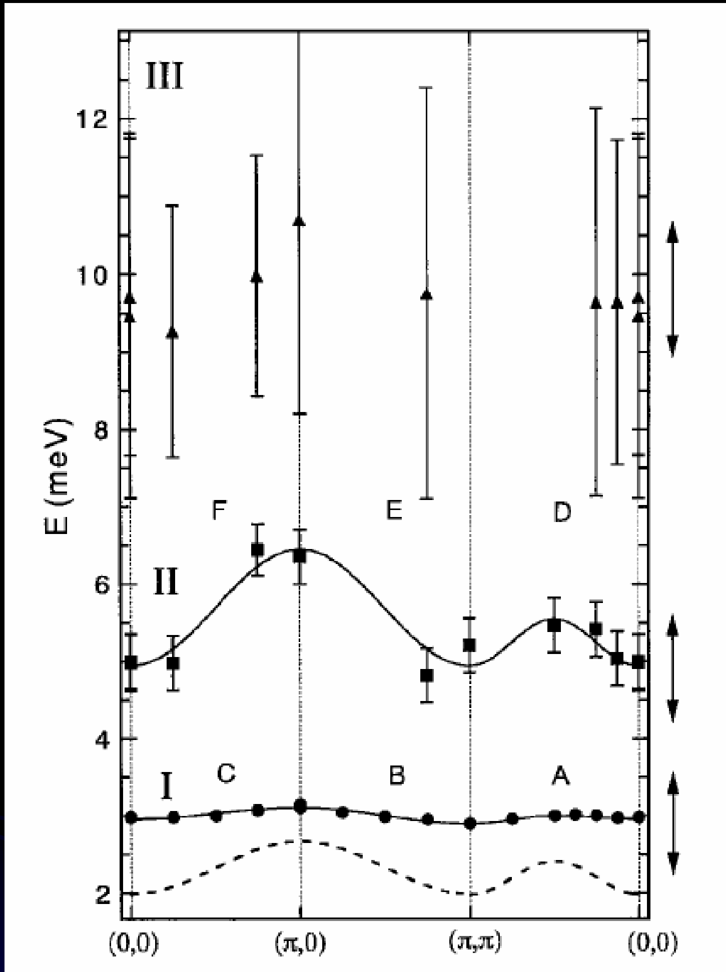
T6: $3^2.4.3.4$ = SrCuBO



T7: 3.4.6.4 = bounce



T8: 3.6.3.6 = kagomé



Triplons on bonds do not propagate well, only pairs do. Massive interacting boson representation is feasible

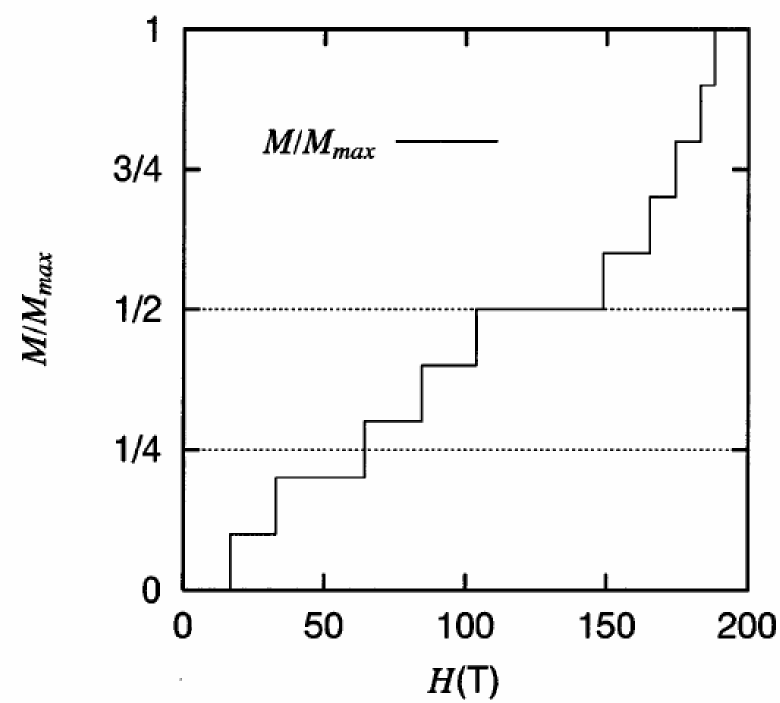


FIG. 5. Magnetization process for the finite cluster of 20 spins. $J = 100$ and $J' = 68$ K are used.

Plateaus: numerical and theoretical + experiments. K Ueda and S Miyahara

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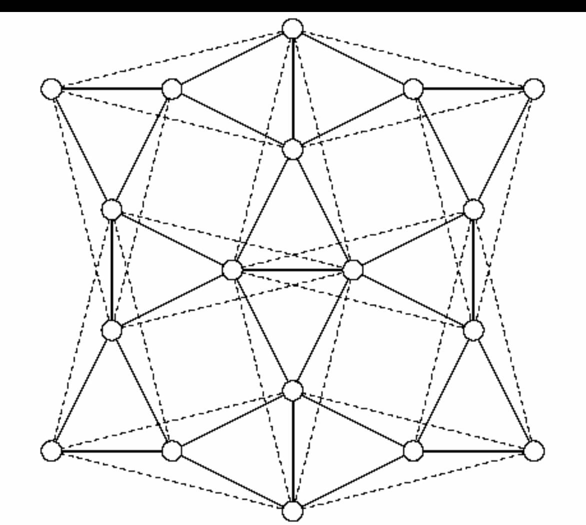
Exact Demonstration of Magnetization Plateaus and First-Order Dimer-Néel Phase Transitions in a Modified Shastry-Sutherland Model for $SrCu_2(BO_3)_2$

Erwin Müller-Hartmann* and Rajiv R. P. Singh†
Institute for Theoretical Physics, University of California, Santa Barbara, California 93106

Christian Knetter and Götz S. Uhrig

$$\mathcal{H} = J_1 \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j + J_2 \sum_{\langle i,k \rangle} \vec{S}_i \cdot \vec{S}_k + J_3 \sum_{\langle i,l \rangle} \vec{S}_i \cdot \vec{S}_l \,, \tag{1}$$



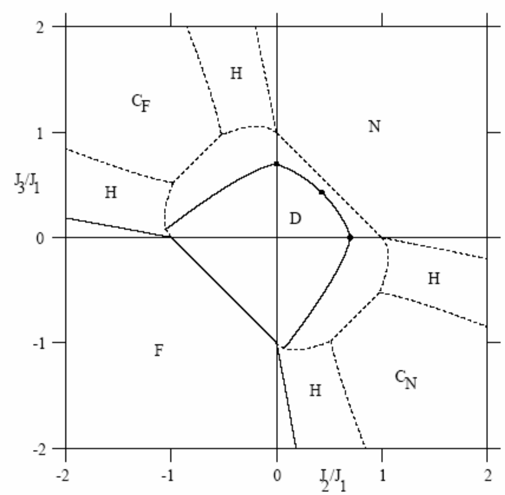


FIG. 2. Phase diagram for the model showing ferromagneti
(F), Néel (N), columnar (C_F and C_N), helical (H), and dime
(D) phases. See text for discussion of phase boundaries.

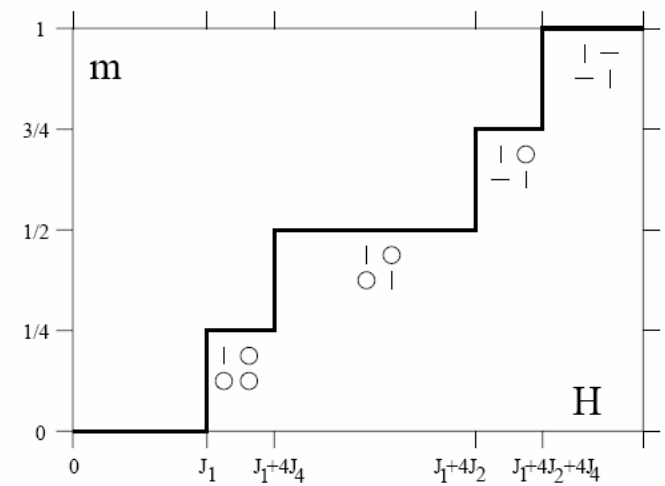


FIG. 4. Magnetization as a function of magnetic field. Note that the plateaus are valid for $J_4 < J_2 < 0.43 J_1$.

VOLUME 87, NUMBER 9 PHYSICAL REVIEW LETTERS 27 AUGUST 2001

Magnetization Plateaus of $SrCu_2(BO_3)_2$ from a Chern-Simons Theory

G. Misguich,[1] Th. Jolicoeur,[2] and S.M. Girvin[3,4]

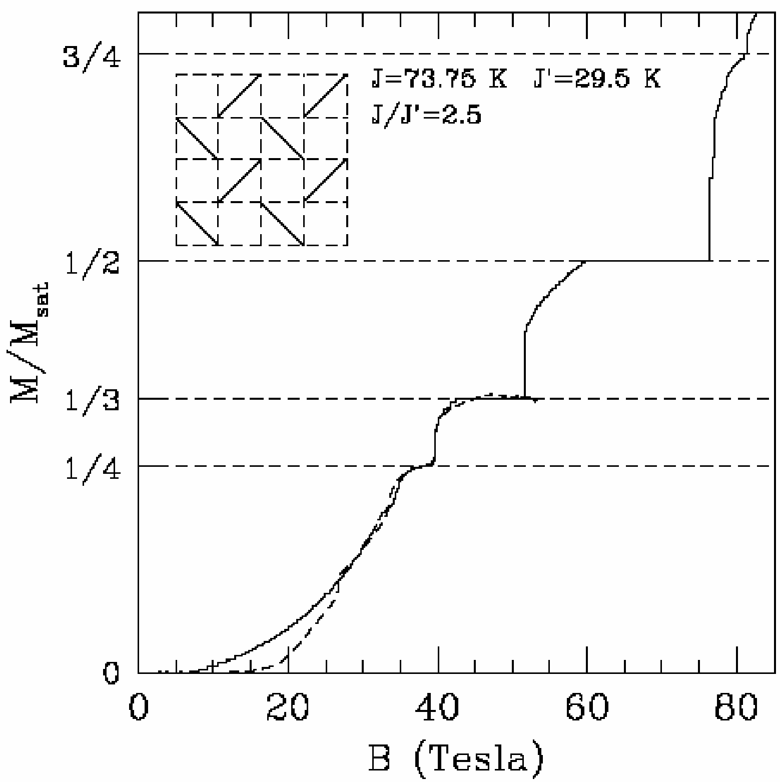
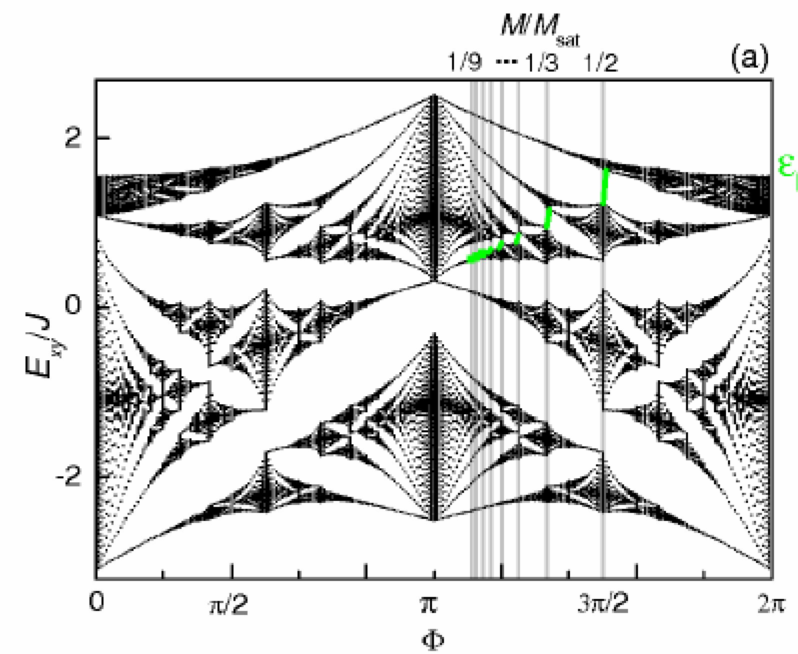


FIG. 1. Comparison between the magnetization curve of $SrCu_2(BO_3)_2$ measured by Onizuka *et al.* (dashed line) and the mean-field result (solid line). Inset: Shastry-Sutherland lattice. The exchange interaction is J on black links and J' on the dotted ones.

et al. in Ref. [3]. By using a Chern-Simons construction on the lattice [27], we can map the hard core bosons in Eq.(1) into spinless fermions: $b_j^\dagger = f_j^\dagger e^{i\sum_{k\neq j}\arg(k,j)n_k}$, $b_j^\dagger = e^{-i\sum_{k\neq j}\arg(k,j)n_k} f_j$ where $arg(k,j)$ is the angle between the relative vector, $\mathbf{r}_k - \mathbf{r}_j$, and an arbitrary direction. This

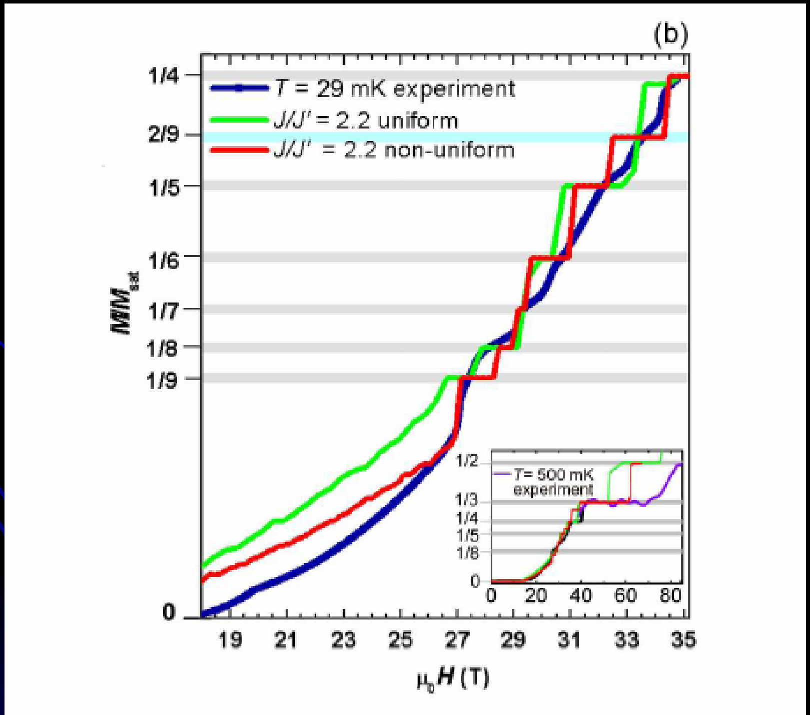


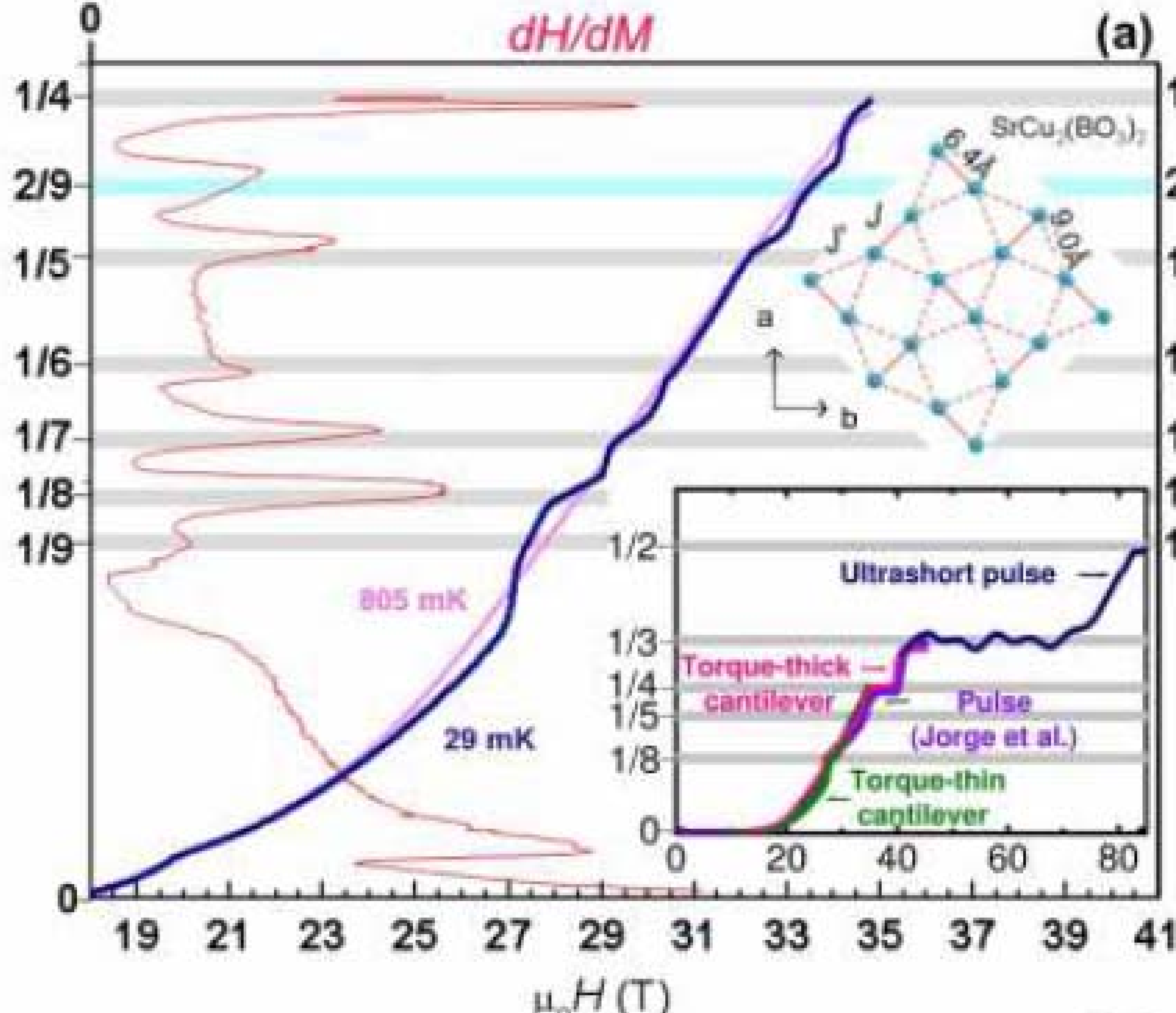
Fractalisation drives crystalline states in a frustrated spin system

Suchitra E. Sebastian,[1] N. Harrison,[2] P. Sengupta,[2,3] C. D. Batista,[3] S. Francoual,[2] E. Palm,[4] T. Murphy,[4] N. Marcano,[1] H. A. Dabkowska,[5] and B. D. Gaulin[5,6]

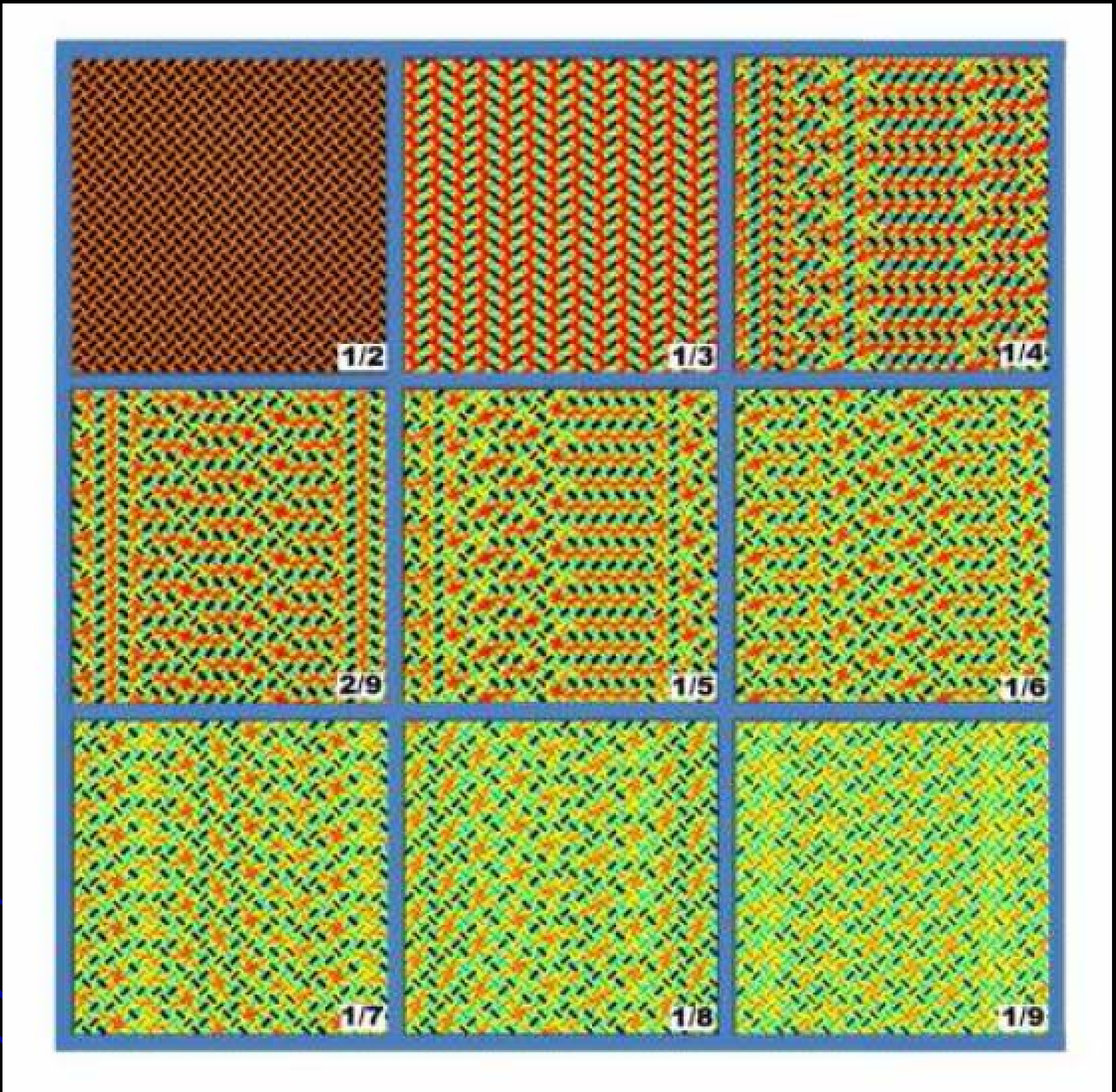
[1] *Cavendish Laboratory, University of Cambridge, Madingley Road, Cambridge CB3 0HE, UK*
[2] *NHMFL, MS-E536, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA*

We measure a sequence of quantum Hall-like plateaux at $1/q$: $9 \geq q \geq 2$ and $p/q = 2/9$ fractions in the magnetisation with increasing magnetic field in the geometrically frustrated spin system $SrCu_2(BO_3)_2$. We find that the entire observed sequence of plateaux is reproduced by solving the Hofstadter problem on the system lattice when short-range repulsive interactions are included, thus providing a sterling demonstration of bosons confined by a magnetic and lattice potential mimicking fermions in the extreme quantum limit.



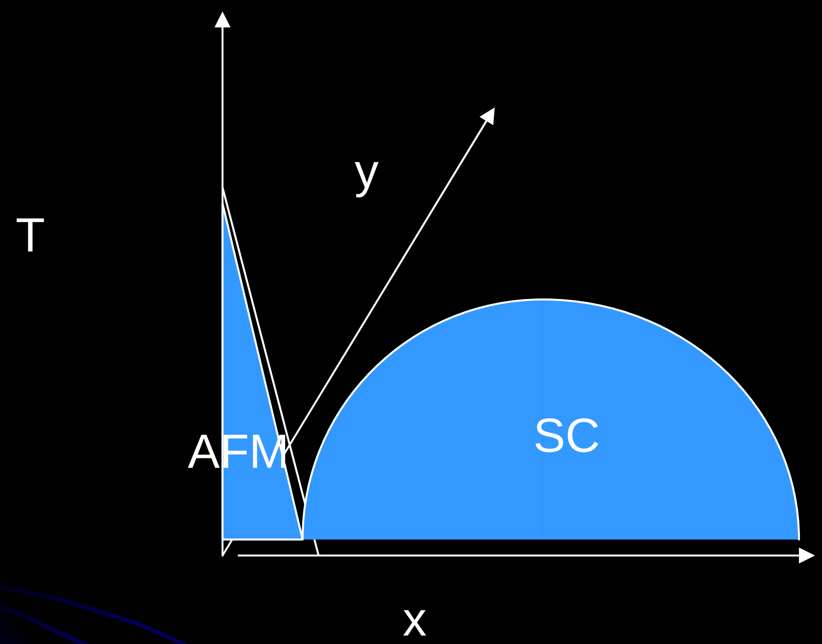


dH/dM
(a)
1/4
2/9
1/5
1/6
1/7
1/8
1/9
0
SrCu2(BO3)2
6.4Å
9.0Å
J'
J
a
b
805 mK
29 mK
M/Msat
1/2
1/3
Ultrashort pulse
Torque-thick cantilever
Pulse (Jorge et al.)
Torque-thin cantilever
0 20 40 60 80
19 21 23 25 27 29 31 33 35 37 39 41
μ0H (T)



“Wigner crystallization” picture of the plateaux phases.

Towards Superconductivity via Mott Phases



AFM state that is a "nuisance" Get rid using a quantum disorder parameter "y" to get a spin liquid

Spin liquid breaks no symmetry, spatial or internal space. E.g. 1-d HAFM Bethe state or 1/r^2 Gutzwiller state

Claim (Shastry Kumar 2002)
SrCu_2(BO_3)_2 is already a spin liquid state, it is a Mott insulator rather than a band insulator, and can be doped to get non trivial electronic superconductivity

$SrCu_2(BO_3)_2$ may usefully be considered as a Mott Insulating state of an underlying Hubbard model. To see this consider the Hubbard model on the SS lattice with

$$H = -t \sum_{\langle i,j \rangle,\sigma} (c^\dagger_{i\sigma} c_{j\sigma} + \text{h.c.}) - \alpha t \sum_{\langle l,m \rangle,\sigma} (c^\dagger_{l,\sigma} c_{m,\sigma} + \text{h.c.}) + U \sum_r n_{r,\uparrow} n_{r,\downarrow}, \qquad (2)$$

Unit cell has 4 Cu atoms therefore by Wilson's rules, is either a band insulator, OR, more interestingly

A semimetal !!

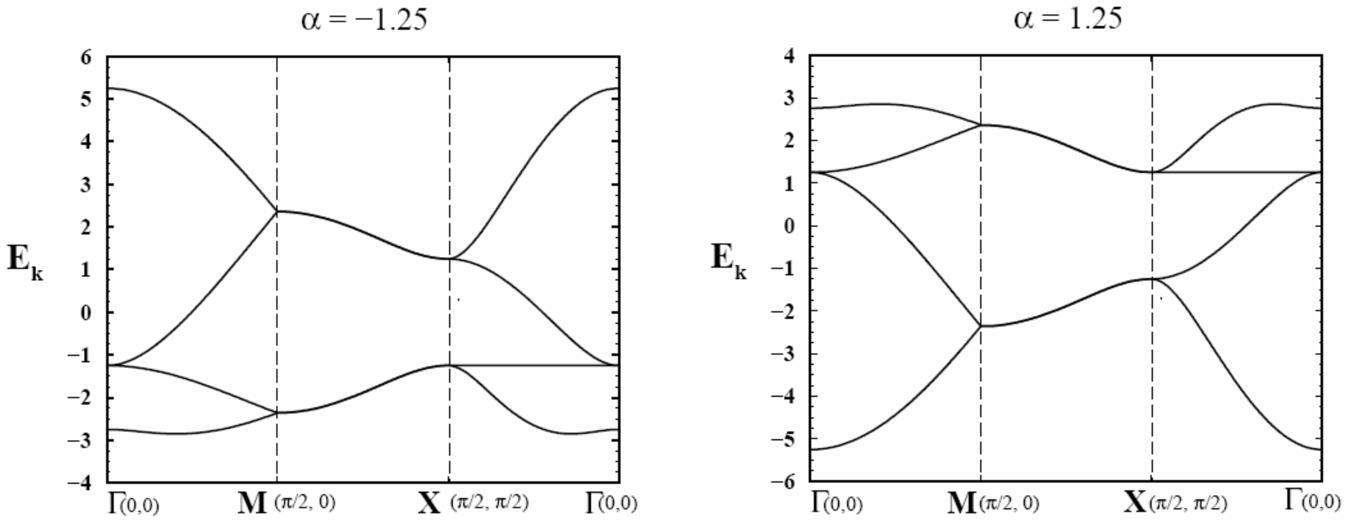


Fig. 4. The band-structure of free electrons on the SS lattice. The wave vectors are written in units of $\frac{1}{a_0}$. Notice that the band-structure is odd with respect to α.

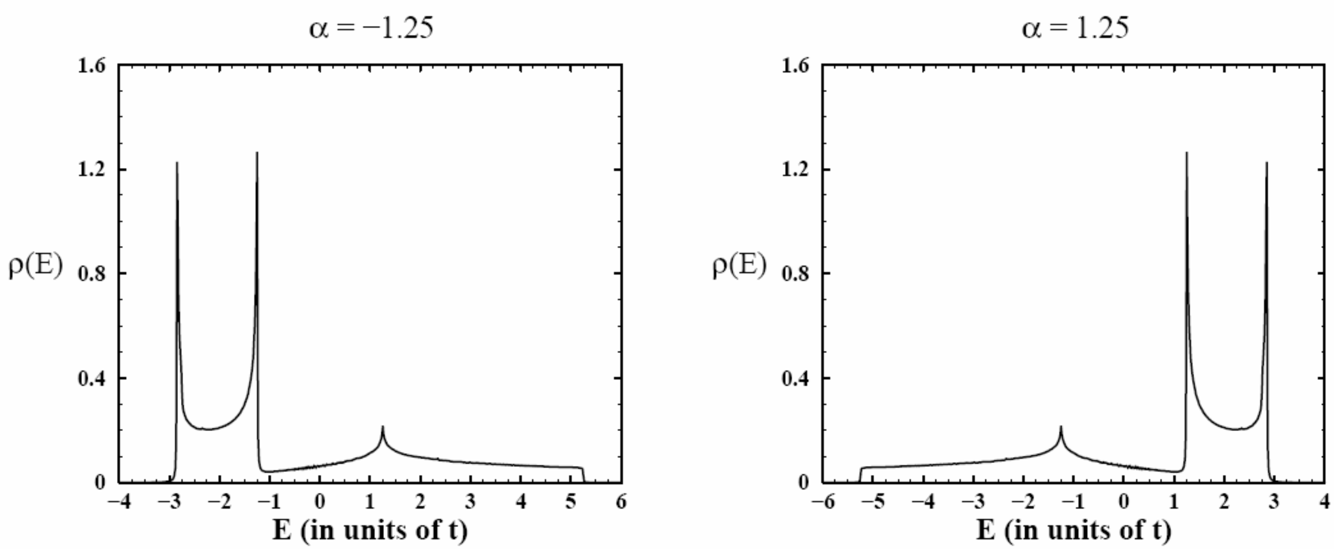


Fig. 5. The density of single-particle electronic states on Shastry-Sutherland lattice.

By solution, it is a semimetal like Graphene.
Infact a strong analogy to bilayer graphene seems to exist,
but with strong interactions (unlike graphene which is uncorrleated to a good appx)

Quadratic touching of filled valence band and empty conduction band, and notice a flat band, either just above or just below the fermi level. Presumably some group theory tells us the inevitability of this pheonmenon. Also possibly adiabatic connected model to bilayer graphene.

Experimentally SrCuBO3 is a good insulator, gap ~ 1 ev. Hence it is a Mott Insulator rather than a band insulator since it breaks no spatial symmetries. It is also a spin liquid state.

Doping Mott states to get superconductors: This is a great experimental challenge. Many groups are trying it. How about theory? What Tc do we expect? Order parameter symmetry?

MFT: Kumar Shastry 2002

More exact Schmalian ..2007

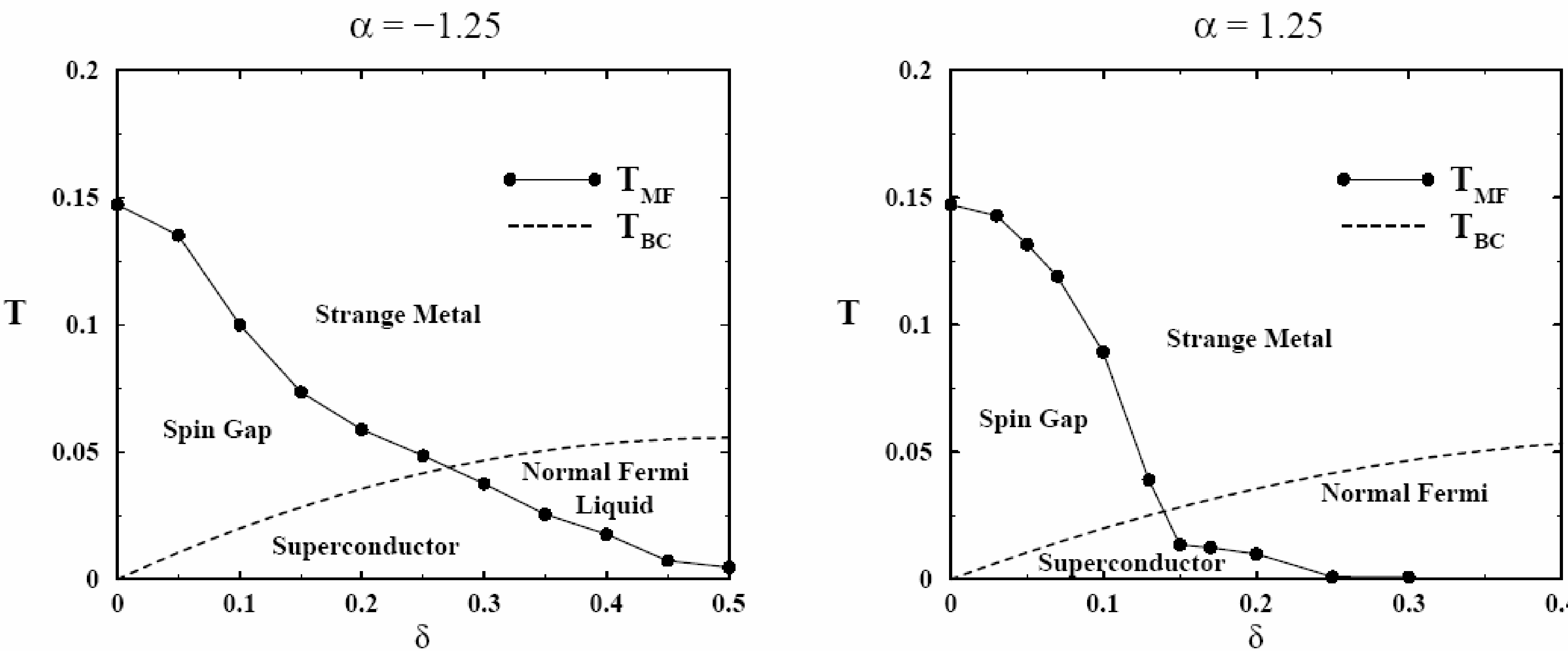


Fig. 7. The phase diagram for the negative as well positive values of α. The lines of the estimated Bose condensation temperature, T_{BC}, and the computed mean field temperature, T_{MF}, divide the T-δ plane into four physically distinct regions. Each of these is appropriately labeled.

In conclusion, the system considered here has a rather rich history. It may also have an important future since under doping it might be the much sought after low T_c RVB superconductor, with linear resistivity down to 10 K and other such exotic properties, rather than a conventional phononic BCS superconductor.

Quantum phases in a doped Mott insulator on the Shastry-Sutherland lattice

Jun Liu,[1] Nandini Trivedi,[2] Yongbin Lee,[1] B. N. Harmon,[1] and Jörg Schmalian[1]

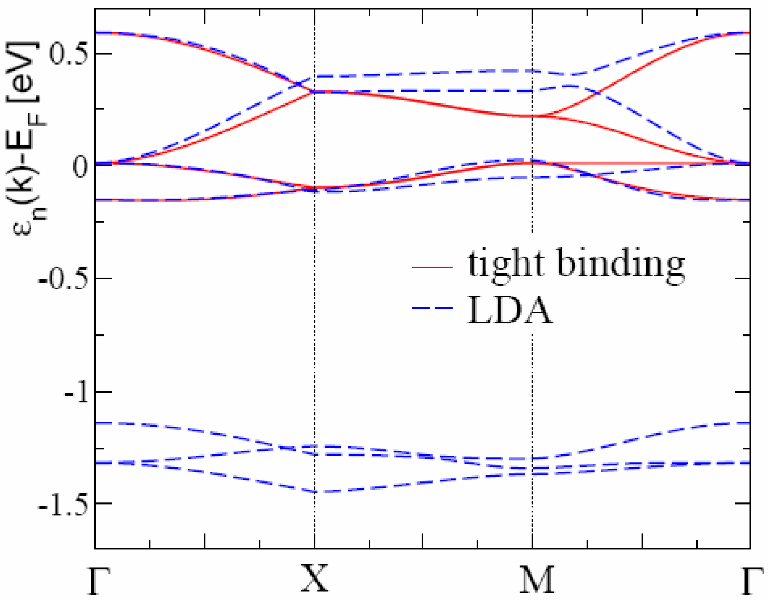
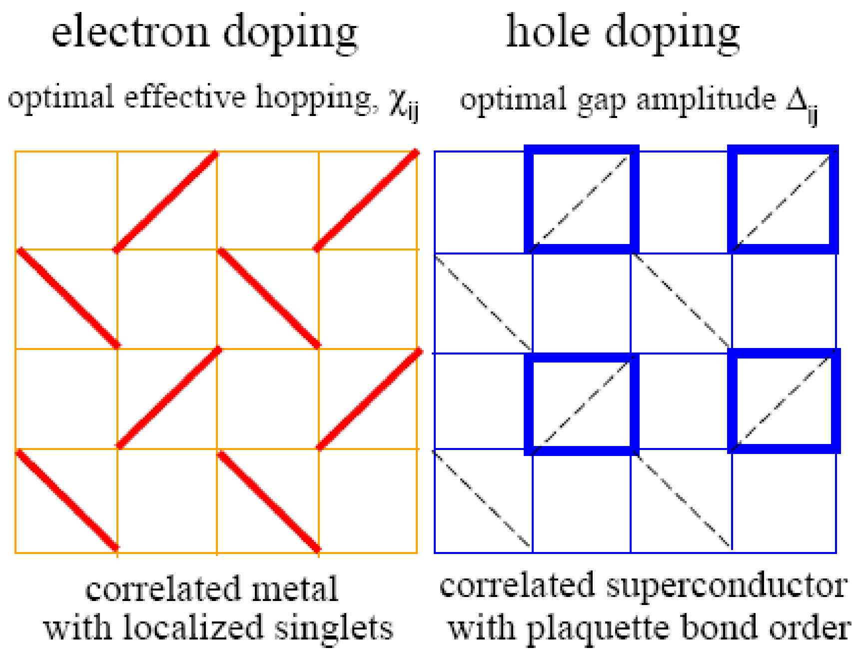
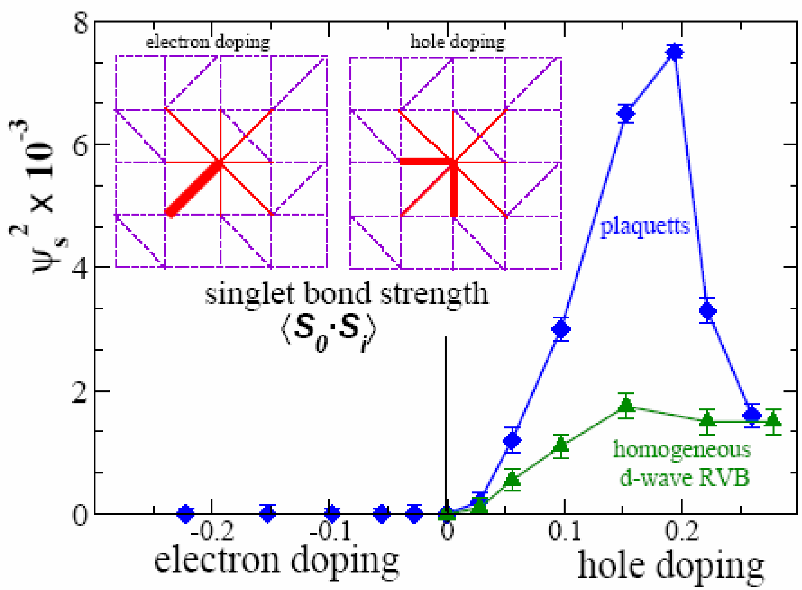


FIG. 2: Bandstructure of $SrCu_2(BO_3)_2$ obtained using density functional theory in comparison with a tight binding fit. Only the in-plane dispersion of the bands is shown. The bands close to E_F are made of Cu-3$d_{x^2-y^2}$ and O-2p states, while the well separated bands around -1.25eV consist of O-2p and Cu-3d_{xz} and 3d_{yz} states.

VMC calculations with projected BCS wave functions. Large scale numerics involved with many variational parameters and also allowed inhomogenous solutions



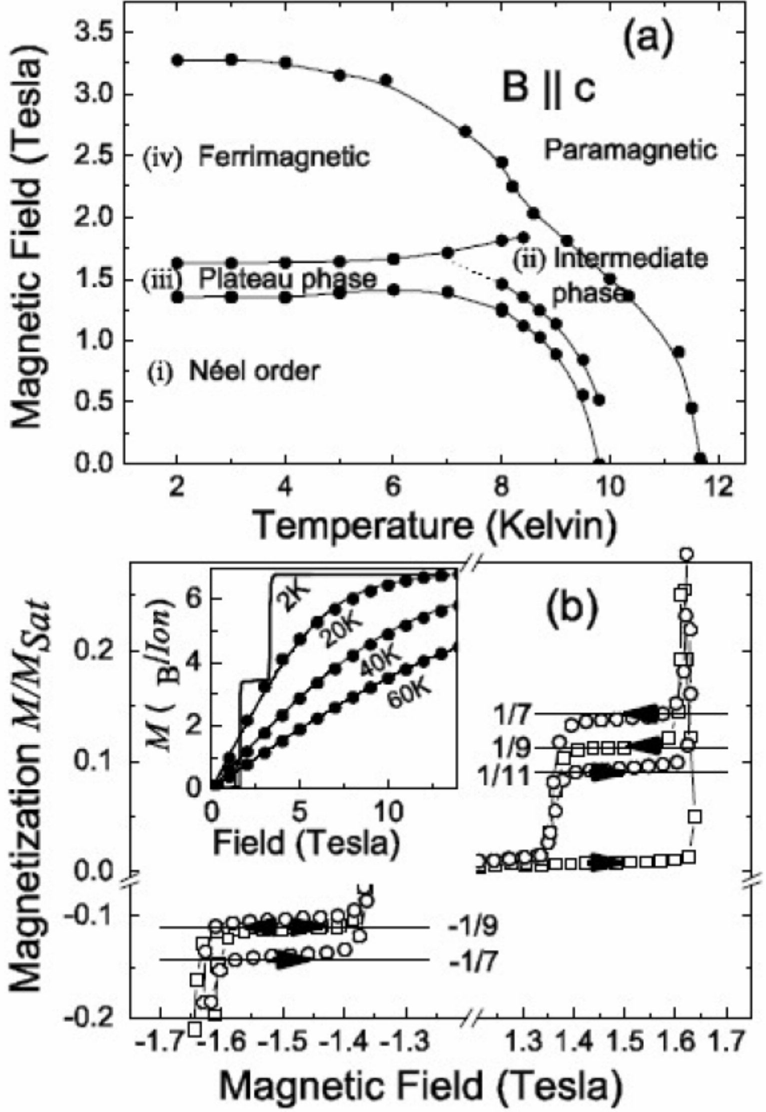
Results are quite symmetric in hole and electron doping. Really interesting system to dope.



Other realizations of SSL

- Rare earth tetraborides (2006-2008) and
- R2T2M

TbB_4	xy	40^0K
DyB_4	$Quadrupolar\ \ order$	$?$
TmB_4	$Isinglike$	11.7^0K
HoB_4	$Isinglike$	7^0K
ErB_4	$Isinglike$	15^0K
Yb_2Pt_2Pb	$mixed(FM+AFM)$	2^0K



Magnetic ordering and fractional magnetization plateaus in the Shastry Sutherland magnet TmB_4

K. Siemensmeyer and E. Wulf
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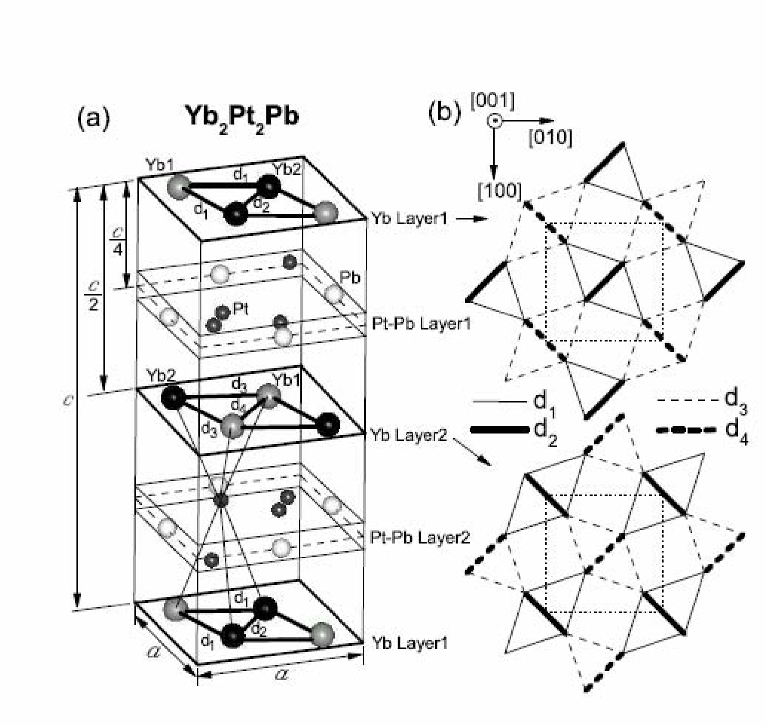
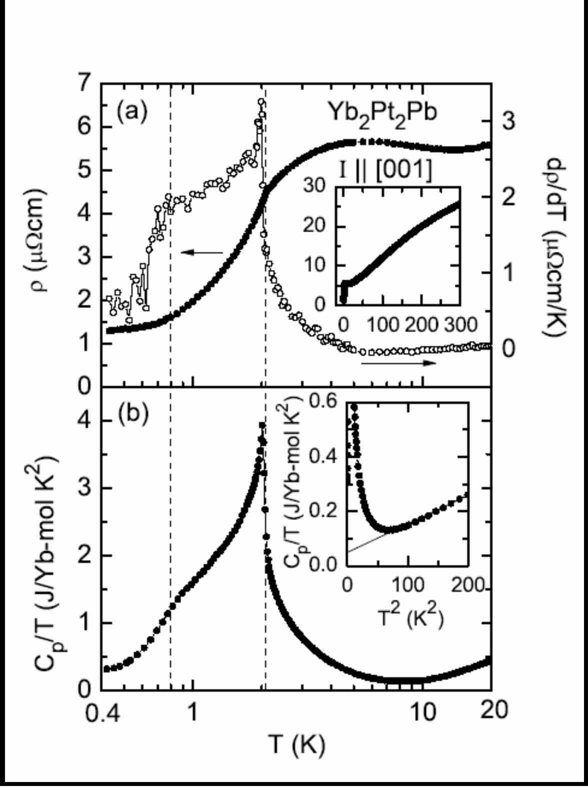
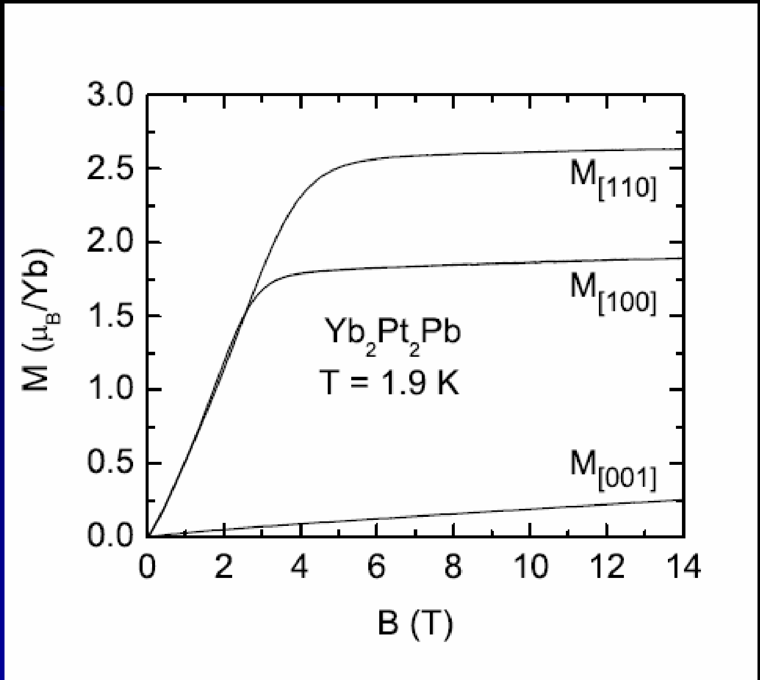
Fig. 1: Fig.1a gives the phase diagram of TmB_4 derived from magnetization data for $B||c$ (full symbols). Fig. 1b shows the magnetization at $T = 2K$ (squares) and $T = 3K$ (circles). The arrows indicate the direction of the field change. The inset to fig. 1b gives





Spin Ice type structure emerges.



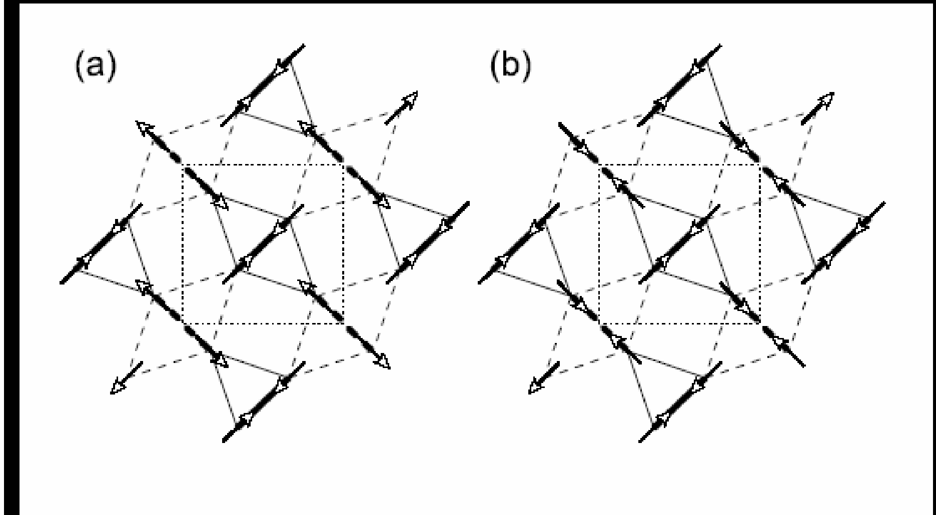


FIG. 7: Schematic representation of an expected magnetic structure on Yb layers. (a) U_2Pt_2Sn type and (b) GdB_4 type.

Yb_2Pt_2Pb: Magnetic frustration in the Shastry-Sutherland lattice

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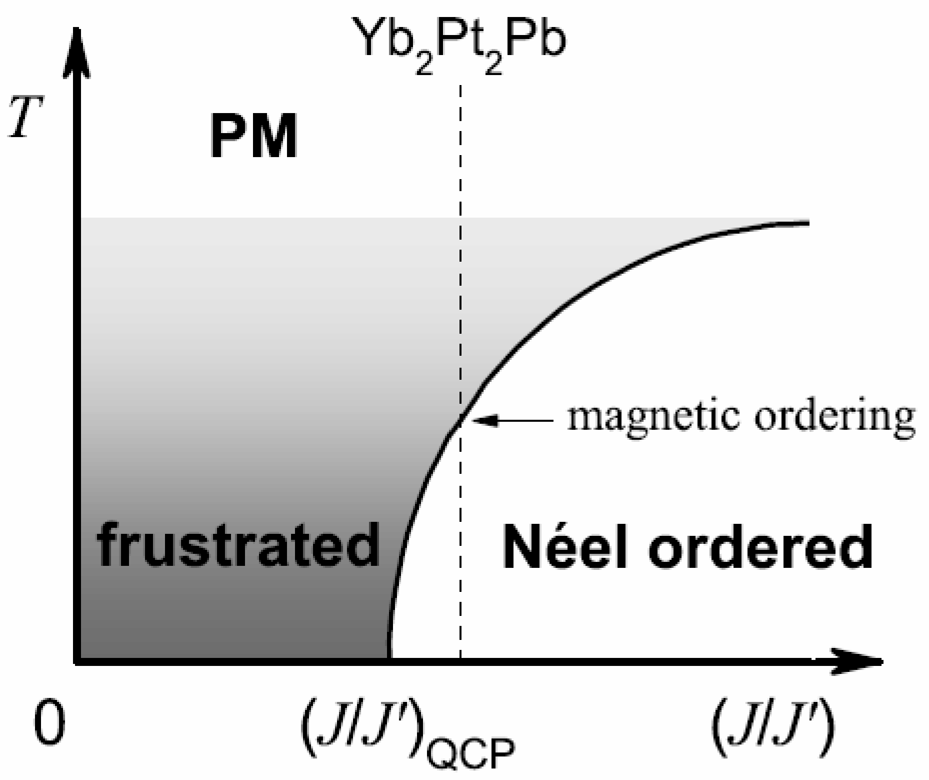

FIG. 8: Schematic phase diagram for the Shastry-Sutherland lattice(see text).

summary

- Variety of magnetic behaviour that is interesting, more frustrated than triangular lattice and tractable in parts.
- Doping is where the biggest fundamental payoff might emerge.
- Meanwhile analogies are rich—bilayer graphene and adiabatic connections.
- Thank you for your attention.