Thermoelectric Effects in Correlated Matter

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High Thermoelectric power is very desirable for applications.

Usually the domain of semiconductor industry, e.g. Bi$_2$Te$_3$. However, recently correlated matter has found its way into this domain.

Heavy Fermi systems (low T), Mott Insulator Junction sandwiches (Harold Hwang 2004)

Sodium Cobaltate NaxCoO2  at $x \sim .7$

Terasaki, Ong, …..
What is the Seebeck Coefficient \( S \)?

\[
\frac{1}{\Omega} \langle \hat{J}_x \rangle = L_{11} E_x + L_{12} (-\nabla_x T/T)
\]

\[
\frac{1}{\Omega} \langle \hat{j}_Q^x \rangle = L_{21} E_x + L_{22} (-\nabla_x T/T),
\]

where \((-\nabla_x T/T)\) is regarded as the \textit{external driving thermal force}, and \( \hat{j}_Q^x \) is the heat current operator.

\[
\text{Thermopower} \quad S(\omega) = \frac{L_{12}(\omega)}{TL_{11}(\omega)}
\]

\[
\text{Lorentz Number} \quad L(\omega) = \frac{\kappa_{zc}(\omega)}{T\sigma(\omega)}
\]

\[
\text{Figure of Merit} \quad Z(\omega) T = \frac{S^2(\omega)}{L(\omega)}. \tag{1}
\]
Desirable: Large ZT

\[ Z \ T = S^2 \frac{T \sigma}{\kappa} \]

- Need large \( S \)
- Large \( \sigma \)
- Small \( \kappa \)

What is \( S \)?

Large variety of answers:

<table>
<thead>
<tr>
<th>Thermodynamic</th>
<th>Kubowallahs</th>
<th>Band Theory .....</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entropy per particle</td>
<td>Kubo formulas</td>
<td>( d \rho(\mu)/d\mu )</td>
</tr>
<tr>
<td>Kelvin</td>
<td>Onsager</td>
<td>Mott</td>
</tr>
</tbody>
</table>
$S_{Kubo} = \left[ \frac{\int_0^\infty dt \int_0^\beta d\tau \langle \hat{J}_x^E (t - i\tau) \hat{J}_x (0) \rangle}{\int_0^\infty dt \int_0^\beta d\tau \langle \hat{J}_x (t - i\tau) \hat{J}_x (0) \rangle} - \frac{\mu(0)}{q_e} \right] + \frac{\mu(0) - \mu(T)}{q_e}$.

$S = $ Transport part + Thermodynamic part Write

$S_{Kubo} = S_{Tr} + S_{Heikes-Mott}$,

Where the first term is the difficult Transport part of $S$.

Similarly thermal conductivity and resitivity are defined with appropriate current operators. The computation of these transport quantities is brutally difficult for correlated systems.

Hence seek an escape route……….That is the rest of the story!
Triangular lattice Hall and Seebeck coeffs: (High frequency objects)

Notice that these variables change sign thrice as a band fills from 0->2. Sign of Mott Hubbard correlations.
Considerable similarity between Hall constant and Seebeck coefficients. Both gives signs of carriers---(Do they actually ???) Zero crossings tell a tale. These objects are sensitive to half filling and hence measure Mott Hubbard hole densities. Brief story of Hall constant to motivate the rest.

The Hall constant at finite frequencies: S Shraiman Singh- 1993
High T_c and triangular lattices---
Consider a novel dispersion relation

(Shastry ArXiv.org 0806.4629)

\[
\Re e R_H(0) = R^*_H(\Omega) + \frac{2}{\pi} \int_0^\Omega \frac{\Im m R_H(\nu)}{\nu} \, d\nu.
\]

• Here \( \Omega \) is a cutoff frequency that determines the \( R^*_H \). LHS is measurable, and the second term on RHS is beginning to be measured (recent data exists).

• The smaller the \( \Omega \), closer is our \( R^*_H \) to the transport value.

• We can calculate \( R^*_H \) much more easily than the transport value.

• For the \( tJ \) model, it would be much closer to the DC than for Hubbard type models. This is obvious since cut off is max\{\( |t|, J| \)\} rather than \( U!! \)

\[
\hbar \omega \gg \{|t|, U\}_{\text{max}}
\]

\[
\hbar \omega \gg \{|t|, J\}_{\text{max}}.
\]
ANALOGY between Hall Constant and Seebeck Coefficients

New Formalism SS (2006) is based on a finite frequency calculation of thermoelectric coefficients. Motivation comes from Hall constant computation (Shastry Shraiman Singh 1993- Kumar Shastry 2003)

\[
\rho_{xy}(\omega) = \frac{\sigma_{xy}(\omega)}{\sigma_{xx}(\omega)^2} \rightarrow BR^*_H \text{ for } \omega \rightarrow \infty
\]

Perhaps \( \omega \) dependence of \( R_H \) is weak compared to that of Hall conductivity.

\[
R^*_H = R_H(0) \text{ in Drude theory}
\]

Very useful formula since

- Captures Lower Hubbard Band physics. This is achieved by using the Gutzwiller projected fermi operators in defining J's.
- Exact in the limit of simple dynamics (e.g few frequencies involved), as in the Boltzmann eqn approach.
- Can compute in various ways for all temperatures (exact diagonalization, high T expansion etc...).
- We have successfully removed the dissipational aspect of Hall constant from this object, and retained the correlations aspect.
- Very good description of t-J model.
- This asymptotic formula usually requires \( \omega \) to be larger than J
Anomalous high-temperature Hall effect on the triangular lattice in Na$_x$CoO$_2$

Yayu Wang$^1$, Nyrissa S. Rogado$^2$, R. J. Cava$^{2,3}$, and N. P. Ong$^{1,3}$

The Hall coefficient $R_H$ of Na$_x$CoO$_2$ ($x = 0.68$) behaves anomalously at high temperatures ($T$). From 200 to 500 K, $R_H$ increases linearly with $T$ to 8 times the expected Drude value, with no sign of saturation. Together with the thermopower $Q$, the behavior of $R_H$ provides firm evidence for strong correlation. We discuss the effect of hopping on a triangular lattice and compare $R_H$ with a recent prediction by Kumar and Shastry.

Hall constant as a function of $T$ for $x=0.68$ (CW metal). $T$ linear over large range 200° to 436° (predicted by theory of triangular lattice transport KS)

STRONG CORRELATIONS & Narrow Bands

T Linear resistivity
Need similar high frequency formulas for $S$ and thermal conductivity.

Requirement::: $L_{ij}(\omega)$

Did not exist, so had lots of fun with Luttinger’s formalism of a gravitational field, now made time dependent.

$$K_{tot} = \sum K(r)(1 + \psi(r, t))$$

$$\nabla(\psi(r, T)) \sim \nabla T(r, t)/T$$
\[
\begin{align*}
&\text{Charge Energy} \\
&I_i \quad J_x(q_x) \quad j_Q^x(q_x) \\
&\mathcal{U}_i \quad \rho(-q_x) \quad K(-q_x) \\
&\mathcal{V}_i \quad E^x_q = iq_x \phi_q \quad i q_x \psi_q.
\end{align*}
\]
We thus see that a knowledge of the three operators gives us a interesting starting point for correlated matter:

High Freq Thermopower  
\[ S^* = \frac{\langle \Phi^{xx} \rangle}{T \langle \tau^{xx} \rangle} \]

High Freq Lorentz Number  
\[ L^* = \frac{\langle \Theta^{xx} \rangle}{T^2 \langle \tau^{xx} \rangle} - (S^*)^2 \]

High Freq Figure of Merit  
\[ Z^* T = \frac{\langle \Phi^{xx} \rangle^2}{\langle \Theta^{xx} \rangle \langle \tau^{xx} \rangle - \langle \Phi^{xx} \rangle^2} \quad (1) \]

\[ \kappa_{zc}(\omega) = \frac{1}{T} \left[ L_{22}(\omega) - \frac{L_{12}(\omega)^2}{L_{11}(\omega)} \right], \]

This leads to interesting sum rules a lè the f-sum rule for conductivity.

\[ \int_{-\infty}^{\infty} \frac{d\nu}{\pi} \Re \kappa_{zc}(\nu) = \frac{1}{T \Omega} \left[ \langle \Theta^{xx} \rangle - \frac{\langle \Phi^{xx} \rangle^2}{\langle \tau^{xx} \rangle} \right]. \]
\[ \int_{-\infty}^{\infty} \frac{d\nu}{2} \text{Re}\sigma(\nu) = \frac{\pi \langle \tau^{xx} \rangle}{2\Omega} \]
\[ \int_{-\infty}^{\infty} \frac{d\nu}{2} \text{Re}\kappa(\nu) = \frac{\pi \langle \Theta^{xx} \rangle}{2T\Omega}, \]

Zero current thermal conductivity where explicit value of \( \mu \) is not needed.
Thermo power operator for Hubbard model

\[ \Phi^{xx} = -\frac{q_e}{2} \sum_{\tilde{\eta},\tilde{\eta}',\tilde{r}} (\eta_x + \eta'_x)^2 t(\tilde{\eta}) t(\tilde{\eta}') \sigma \frac{c^+_x}{\tilde{r} + \tilde{\eta} + \tilde{\eta}',\sigma} c_{\tilde{r},\sigma} - q_e \mu \sum_{\tilde{\eta}} \eta_x t(\tilde{\eta}) \sigma \frac{c^+_x}{\tilde{r} + \tilde{\eta},\sigma} c_{\tilde{r},\sigma} + \]

\[ q_e U \frac{t(\tilde{\eta}) (\eta_x)^2}{4} \sum_{\tilde{r},\tilde{\eta}} (n_{\tilde{r},\sigma} + n_{\tilde{r} + \tilde{\eta},\sigma})(c^+_x \frac{\sigma}{\tilde{r} + \tilde{\eta},\sigma} c_{\tilde{r},\sigma} + c^+_x \frac{\sigma}{\tilde{r},\sigma} c_{\tilde{r} + \tilde{\eta},\sigma}). \]

This object can be expressed completely in Fourier space as

\[ \Phi^{xx} = q_e \sum_{\vec{p}} \frac{\partial}{\partial p_x} \left\{ v^x_p (\varepsilon_{\vec{p}} - \mu) \right\} \frac{c^+_x}{\vec{p},\sigma} c_{\vec{p},\sigma} \]

\[ + \frac{q_e U}{2L} \sum_{\tilde{l},\tilde{p},\tilde{q},\sigma,\sigma'} \frac{\partial^2}{\partial l^2_x} \left\{ \varepsilon_{\tilde{l}} + \varepsilon_{\tilde{l} + \tilde{q}} \right\} \frac{c^+_x}{\tilde{l} + \tilde{q},\sigma} c_{\tilde{l},\sigma} \frac{c^+_x}{\tilde{p} - \tilde{q},\sigma'} c_{\tilde{p},\sigma'}. \]

\[ \tau^{xx} = \frac{q^2_e}{\hbar} \sum_{\tilde{\eta}_x^2} t(\tilde{\eta}) \frac{c^+_x}{\tilde{r} + \tilde{\eta},\sigma} c_{\tilde{r},\sigma} \text{ or} \]

\[ \tau^{xx} = \frac{q^2_e}{\hbar} \sum_{\tilde{k}} \frac{d^2 \varepsilon_{\tilde{k}}}{dk^2_x} \frac{c^+_x}{\tilde{k},\sigma} c_{\tilde{k},\sigma} \]
\[ \Theta^{xx} = \sum_{p, \sigma} \frac{\partial}{\partial p_x} \{ v_p^x (\varepsilon_p - \mu)^2 \} \ c_{\bar{p}, \sigma}^+ c_{\bar{p}, \sigma} + \frac{U^2}{4} \sum_{\eta, \sigma} t(\bar{\eta}) \eta_x^2 (n_{\tilde{r}, \bar{\sigma}} + n_{\tilde{r}+\bar{\eta}, \bar{\sigma}})^2 c_{\tilde{r}+\bar{\eta}, \sigma}^+ c_{\tilde{r}, \sigma} \]

\[ -\mu U \sum_{\eta, \sigma} t(\bar{\eta}) \eta_x^2 (n_{\tilde{r}, \bar{\sigma}} + n_{\tilde{r}+\bar{\eta}, \bar{\sigma}}) c_{\tilde{r}+\bar{\eta}, \sigma}^+ c_{\tilde{r}, \sigma} \]

\[ -\frac{U}{8} \sum_{\tilde{\eta}, \tilde{\eta}', \sigma} t(\tilde{\eta}) t(\tilde{\eta}') (\eta_x + \eta_x')^2 \left\{ 3n_{\tilde{r}, \bar{\sigma}} + n_{\tilde{r}+\tilde{\eta}, \bar{\sigma}} + n_{\tilde{r}+\tilde{\eta}', \bar{\sigma}} + 3n_{\tilde{r}+\tilde{\eta}+\tilde{\eta}', \bar{\sigma}} \right\} c_{\tilde{r}+\tilde{\eta}+\tilde{\eta}', \sigma}^+ c_{\tilde{r}, \sigma} \]

\[ +\frac{U}{4} \sum_{\tilde{\eta}, \tilde{\eta}', \sigma} t(\tilde{\eta}) t(\tilde{\eta}') (\eta_x + \eta_x') \eta_x^2 c_{\tilde{r}+\tilde{\eta}, \sigma}^+ c_{\tilde{r}, \sigma} \left\{ c_{\tilde{r}+\tilde{\eta}, \bar{\sigma}}^+ c_{\tilde{r}+\tilde{\eta}+\tilde{\eta}', \bar{\sigma}} + c_{\tilde{r}-\tilde{\eta}', \bar{\sigma}}^+ c_{\tilde{r}, \bar{\sigma}} - h.c. \right\}. \quad (1) \]

Unpublished- For Hubbard model using “Ward type identity” can show a simpler result for \[\Phi\].

\[ \langle \Phi^{xx} \rangle = \frac{q_e}{c} k_B T \sum_{m, \sigma, \bar{k}} G_{\sigma} (k, i\omega_m) \left[ \frac{d}{dk_x} (v_k^x (\varepsilon_k - \mu)) + \frac{d^2 \varepsilon_k}{dk_x^2} \sum_{\sigma} (k, i\omega_m) \right] \]
New Formalism:

• Novel way for computing thermopower of isolated system (absolute Thermopower)
• Leads to correct Onsager formula (a la Kubo)
• Leads to other insights and other useful formulae
• Settles the Kelvin- Onsager debate.
  • Kelvin derived reciprocity between Peltier and Seebeck Coefficient using only thermodynamics,
  • Onsager insisted that Dynamics is needed to establish reciprocity.
  • According to Wannier’s book on Statistical Physics “Opinions are divided on whether Kelvin’s derivation is fundamentally correct or not”.

Use Luttinger’s technique

Turn on spatially inhomogeneous time dependent potential adiabatically from remote past.

\[ H_1 = \sum_j H(r_j) \psi(r_j) \exp(\eta - i\omega)t \]

\[ H_1 \sim \sum_j \frac{H(r_j)}{c^2} V_{\text{gravity}}(r_j) \]

System of Length L, open at the two surfaces 1 and 2

\[ T_1 \quad \text{Sample along x axis} \quad T_2 \]

Choose linear gravitational potential \( \psi(r_j) = \frac{x}{L} \psi_0 \).

Fundamental theorem of Luttinger: Gravitational pot \( \sim \) temperature. More precisely: in a suitable limit

\[ \nabla \psi(r) \rightarrow \frac{\nabla T(r)}{T} \]
1. Compute the induced change in particle density profile:

2. Particles run away from hot end to cold end, hence pileup a charge imbalance i.e. a dipole moment:

3. Linear response theory gives the dipole moment amplitude:

\[
P_{\text{Thermal}} = \sum_r \langle x n(r) \rangle \exp i \omega t
\]

\[
= q e \frac{T_2 - T_1}{T} \chi[\sum x n(r), \sum x (H(r) - \mu n(r))] (\omega)
\]

Identical calculation with electrostatic potential gives:

\[
P_{\text{Elec}} = q e (\phi_2 - \phi_1) \chi[\sum x n(r), \sum x n(r)] (\omega)
\]

Hence: the thermo-power is obtained by asking for the ratio of forces that produce the same dipole moment! (balance condition)

\[
S = \frac{\phi_1 - \phi_2}{T_1 - T_2}
\]

\[
= \frac{q e \chi[\sum x n(r), \sum x (H(r) - \mu n(r))] (\omega)}{T \chi[\sum x n(r), \sum x n(r)] (\omega)}
\]

Susceptibility of two measurables A, B is written as

\[
\chi[A,B] (\omega)
\]
What about Kelvin-Onsager?

\[
S = \lim_{\omega \to 0, q_x \to 0} \frac{1}{q e T} \frac{\chi[n_q, H - q - \mu n_q](\omega)}{\chi[n_q, H - q - \mu n_q](\omega)}
\]

Onsager-Kubo

Large box then static limit

\[
S_{\text{Kelvin}} = \lim_{q_x \to 0, \omega \to 0} \frac{1}{q e T} \frac{\chi[n_q, H - q - \mu n_q](\omega)}{\chi[n_q, H - q - \mu n_q](\omega)}
\]

Kelvin Thermodynamics

Static limit then large box

\[
S^* = \lim_{\omega \gg \omega_c, q_x \to 0} \frac{1}{q e T} \frac{\chi[n_q, H - q - \mu n_q](\omega)}{\chi[n_q, H - q - \mu n_q](\omega)}
\]

High Frequency

Large box then frequency larger than characteristic \( w \)’s
For a weakly interacting diffusive metal, we can compute all three S's. Here is the result:

Density Of States

\[ S = T \frac{\pi^2 k_B^2}{3 q_e} \frac{d}{d \varepsilon} \ln \left[ \rho(\varepsilon) \langle (v^x)^2 \rangle_{\varepsilon} \tau(\varepsilon) \right]_{\varepsilon \rightarrow \mu} \]  
Onsager-Kubo-Mott formula

Velocity averaged over FS

\[ S = T \frac{\pi^2 k_B^2}{3 q_e} \frac{d}{d \varepsilon} \ln \left[ \rho(\varepsilon) \right]_{\varepsilon \rightarrow \mu} \]  
Kelvin inspired formula

Energy dependent relaxation time.

\[ S^* = T \frac{\pi^2 k_B^2}{3 q_e} \frac{d}{d \varepsilon} \ln \left[ \rho(\varepsilon) \langle (v^x)^2 \rangle_{\varepsilon} \right]_{\varepsilon \rightarrow \mu} \]  
High frequency formula

Easy to compute for correlated systems, since transport is simplified!
Theorem:
Kelvin inspired formula is the best possible thermodynamic approximation to Onsager Kubo transport formula

Some results: M Peterson SS 2008 Unpublished

Unflipped-NCO

Flipped- Fiduciary hole doped CO$_2$
And now for some results:

Triangular lattice t-J exact diagonalization (full spectrum)

Collaboration and hard work by:-

J Haerter, M. Peterson, S. Mukerjee (UC Berkeley)
How good is the $S^*$ formula compared to exact Kubo formula?

A numerical benchmark: Max deviation 3\% anywhere!!

As good as exact!

$x=0.67$, $t>0$, $J=0.2|t|$
Results from this formalism:

Strong Correlations Produce the Curie-Weiss Phase of Na$_x$CoO$_2$

Jan O. Haerter, Michael R. Peterson, and B. Sriram Shastry

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(Received 21 July 2006; published 28 November 2006)

T linear Hall constant for triangular lattice predicted in 1993 by Shastry Shraiman Singh! Quantitative agreement hard to get with scale of “t”

Comparison with data on absolute scale!

Prediction for t>0 material
$S^*$ and the Heikes Mott formula (red) for Na$_x$Co O$_2$. Close to each other for $t>0$ i.e. electron doped cases

$t>0, \ J=0.2|t|$
Predicted result for $t<0$ i.e. fiducary hole doped $\text{CoO}_2$ planes. Notice much larger scale of $S^*$ arising from transport part (not Mott Heikes part!!).

Enhancement due to triangular lattice structure of closed loops!! Similar to Hall constant linear $T$ origin.
Predicted result for $t<0$ i.e. fiducary hole doped CoO$_2$ planes.

Different $J$ higher $S$. 

$t<0$, $J=40$ K
Predictions of $S^*$ and the Heikes Mott formula (red) for fiducary hole doped CoO$_2$.

Notice that $S^*$ predicts an important enhancement unlike Heikes Mott formula.
$Z^*T$ computed from $S^*$ and Lorentz number. Electronic contribution only, no phonons. Clearly large $x$ is better!!

Quite encouraging.
Phenomenological eqns for coupled charge heat transport

- Meaning of the new operators becomes clear.
- Some interesting experiments using laser heating are suggested.

\[
\left[ \frac{1}{\tau} + \frac{d}{dt} \right] \langle \hat{J}_x^Q (\vec{r}, t) \rangle \quad = \quad - \frac{D_Q}{\tau} \nabla \langle K(\vec{r}t) \rangle - \frac{c_1}{\tau} \nabla \langle \rho(\vec{r}t) \rangle \\
- \left\{ \frac{\langle \Theta^{xx} \rangle_0}{\Omega} \nabla \psi(\vec{r}t) + \frac{\langle \Phi^{xx} \rangle_0}{\Omega} \nabla \phi(\vec{r}t) \right\}
\]

(1)

and

\[
\left[ \frac{1}{\tau} + \frac{d}{dt} \right] \langle \hat{J}_x (\vec{r}, t) \rangle \quad = \quad - \frac{c_2}{\tau} \nabla \langle K(\vec{r}t) \rangle - \frac{D_c}{\tau} \nabla \langle \rho(\vec{r}t) \rangle \\
- \left\{ \frac{\langle \tau^{xx} \rangle_0}{\Omega} \nabla \phi(\vec{r}t) + \frac{\langle \Phi^{xx} \rangle_0}{\Omega} \nabla \psi(\vec{r}t) \right\}
\]

(2)
Hydrodynamics of energy and charge transport in a band model:
This involves the fundamental operators in a crucial way:

Continuity

\[ \frac{\partial \rho}{\partial t} + \nabla J(r) = 0 \]

\[ \frac{\partial K(r)}{\partial t} + \nabla J^Q(r) = p_0 \delta(x) \]

These eqns contain energy and charge diffusion, as well as thermoelectric effects. Potentially correct starting point for many new nano heating expts with lasers. Work in progress. Preprint soon
Hydrodynamics of energy and charge transport in a band model:
This involves the fundamental operators in a crucial way:

\[
\left\{ \frac{\partial}{\partial t} + \frac{1}{\tau_c} \right\} \delta J(r) = \frac{1}{\Omega} \langle \tau^{xx} \rangle \left[ \frac{1}{q_e^2} \frac{\partial \mu}{\partial n} \left( -\nabla \rho - \nabla \phi(r) \right) \right] + \frac{1}{\Omega} \langle \Phi^{xx} \rangle \left[ \frac{1}{C(T)} \left( -\nabla K(r) \right) - \nabla \Psi \right]
\]

\[
\left\{ \frac{\partial}{\partial t} + \frac{1}{\tau_E} \right\} \delta J^Q(r) = \frac{1}{\Omega} \langle \Phi^{xx} \rangle \left[ \frac{1}{q_e^2} \frac{\partial \mu}{\partial n} \left( -\nabla \rho - \nabla \phi(r) \right) \right] + \frac{1}{\Omega} \langle \Theta^{xx} \rangle \left[ \frac{1}{C(T)} \left( -\nabla K(r) \right) - \nabla \Psi \right]
\]

\[
\frac{\partial \rho}{\partial t} + \nabla J(r) = 0
\]

\[
\frac{\partial K(r)}{\partial t} + \nabla J^Q(r) = p_{ext}(r)
\]

Einstein diffusion term of charge
Energy diffusion term
Continuity
Input power density

These eqns contain energy and charge diffusion, as well as thermoelectric effects. Potentially correct starting point for many new nano heating expts with lasers.
Some results from the coupled charge–energy (entropy) hydrodynamics:

$$N_2 = \frac{1}{A} \partial \delta K_q = \frac{\partial \delta K_q}{\partial P_0}$$

$$\delta J^Q_x = L_{22}(iq\psi_q) + L_{21}(iq\phi_q) + M_2 \frac{P_0}{L}$$

$$\Delta = (1 - i\omega\tau + i\frac{D_Q q^2}{\omega})(1 - i\omega\tau + i\frac{D_c q^2}{\omega}) + \xi D_c D_Q \frac{q^4}{\omega^2},$$

$$\xi = \frac{\langle \Phi^{xx} \rangle_0^2}{\langle \Theta^{x x} \rangle_0} = \frac{Z^* T}{Z^* T + 1}$$

$$L_{22} = \frac{1}{\Delta} \frac{\tau \langle \Theta^{x x} \rangle_0}{\Omega} \left[ 1 - i\omega\tau + i(1 - \xi) D_c \frac{q^2}{\omega} \right]$$

$$M_2 = -\frac{1}{\Delta} D_Q \frac{q}{\omega} \left[ 1 - i\omega\tau + i(1 - \xi) D_c \frac{q^2}{\omega} \right].$$

$$N_2 = \frac{i}{\omega} - \frac{q}{\omega} M_2$$
\[ \delta K(t, 0) = \delta K^+(t, 0) + \delta K^-(t, 0) \]

\[
\delta K^+(t, 0) = \frac{2}{\pi} P_0 q_0 \left( e^{-\frac{t}{2\tau_q}} \left[ \frac{R_f}{2\tau_q} \right] + \frac{sh \left[ \frac{R_f}{2\tau_q} \right]}{R_q} \right) dq
\]

\[
\delta K^-(t, 0) = \frac{2}{\pi} P_0 q_0 \left( e^{-\frac{t}{2\tau_q}} \left[ \cos \left( \frac{R_f}{2\tau_q} \right) + \frac{\sin \left[ \frac{R_f}{2\tau_q} \right]}{R_q} \right] \right) dq
\]

\[
q_m = \frac{\pi}{a}; q_0 = \frac{1}{2\sqrt{D_0 \tau_0}}; R_q = \sqrt{1 - 4D_0 \tau_0 q^2}; \text{and } \bar{R}_q = \sqrt{4D_0 \tau_0 q^2 - 1}
\]
Conclusions

- New insights into thermal transport and thermoelectric effects - predictions for new material design.
- Novel framework with new objects
- Simple minded hydrodynamics gives a useful picture of the transport phenomena, as an alternative to usual transport Boltzmann eqns.