Physics 101a HW # 8: Due Friday, Dec. 3, 04

(I put a lot of thought into this assignment and I think it may be very valuable in solidifying both your technical and conceptual understanding of quantum physics. Please post questions, comments and corrections to the ERES discussion board.)

1. Consider an electron in a finite square well:

$V = V_0$ for $x \geq L/2$,

$V = 0$ for $-L/2 < x < L/2$,

$V = V_0$ for $x \leq -L/2$,

a) For $V_0 \approx 4$ eV and L= 1 nm how many bound states does the well have?

b) Sketch the well, and in that well-sketch, show dashed horizontal lines at roughly the energy of the ground state, the 1st excited state and the 2nd excited state, E_1, E_2, E_3.

c) Sketch the wavefunction $\psi(x)$ for the ground state, the 1st excited state and the 2nd excited state (ψ_1, ψ_2, ψ_3).

d) Roughly estimate the exponential decay length scale for each of these three states. (This is not asking for $< x^2 >^{1/2}$. Just estimate α^{-1}, {or $(2\alpha)^{-1}$ if you prefer} for each of these 3 states.) Which is largest? How do they compare with the width of the well?

e) *Important!* If you double V_0, how does that effect α^{-1} for these states?

2. For the potential and the *ground state* wavefunction from problem 1):

a) Write an expression for the (expectation value of the) potential energy in terms of definite integrals. Don't try to evaluate them, just write down what the integrals are. Your answer should be in the form of 3 integrals (or two if you use symmetry). The integrands should involve undetermined coefficients A, B, k and α .

b) Imagining that you have already determined k and α write the integrals that express the normalization of the ground state, i.e., $1 = ... + ... + ...$ Again, your answer should be in the form of 3 integrals (or two if you use symmetry. okay maybe just one that's non-zero)).

c) Guess which of your 3 states from problem 1 has the highest and lowest potential energy and explain your guess. [See below for extra credit.]

3. Consider a square well with a barrier in the middle:

$V = V_0$ for $x \geq d + L$,

$V = 0$ for $d < x < d + L$,

$V = V_1$ for $-d < x < d$,

$V = 0$ for $-(d + L) < x < -d$,

$V = V_0$ for $x \leq -(d + L)$,

a) Sketch the well for $V_0 \approx 2$ eV , $V_1 \approx 3$ eV, and L= 1 nm and 2d=0.2 nm.

b) Sketch what you would imagine the ground state and 1st excited state look like for the extreme cases: i) $V_1 = 0$, and the $V_1 \to \infty$

c) Sketch what you would imagine the ground state and 1st excited state look like for the intermediate case $V_1 \approx 3$ eV.

d) Estimate the energy difference between the ground state and first excited state, $E_2 - E_1$, for the extreme cases: i) $V_1 = 0$, and the $V_1 \to \infty$

e) Use your results from d, do a very rough sketch of $E_2 - E_1$ vs the height of the barrier, V_1. You don't need to try to put a horizontal scale on this graph (though see f).

f) Extra credit (very difficult): Estimate the crossover energy scale for part e). (That is, at roughly what energy scale does the value of $E_2 - E_1$ undergo most rapid change?) This is not easy. It may be do-able with the aid of a webb site or numerical methods.

4. Consider a harmonic oscillator potential with a barrier in the middle. i.e.:
$V = 1/2kx^2$ for $x \geq d$; $V = V_1$ for $-d < x < d$; $V = 1/2kx^2$ for $x \leq -d$.
Suppose that $k = m\omega^2$ and $\hbar\omega = 1$ eV.

a) Sketch the well for $V_1 \approx 2$ eV 2d=0.2 nm.

b) Calculate the width of the ground state, $< x^2 >^{1/2}$, for the case of $V_1 = 0$ (no barrier)? (Integrate, and get a result in nm for this case.) How is it related to $(\hbar/(m\omega))^{1/2}$? Is it greater than or smaller than d=0.2 nm?

c) Sketch what you would imagine the ground state and 1st excited state look like for the extreme cases: i) $V_1 = 0$, and the $V_1 \to \infty$.

d) Sketch what you would imagine these states look like for an intermediate case.

e) Estimate the energy difference between the ground state and first excited state, $E_2 - E_1$, for the extreme cases: i) $V_1 = 0$, and the $V_1 \to \infty$ [Note that E_1 refers to the ground state!]

f) Do a rough sketch of $E_2 - E_1$ vs V_1. Skip the horizontal scale on this graph (too hard).

5. This problem deals with the absorption of light by a confined electron. This occurs via transitions between states. Like hydrogen, these wells will have *"spectroscopic signatures"* associated with transitions between quantum states of the electron. These problems ask for graphs of absorption of light as a function of photon frequency f. Don't worry about the vertical scale. (That's not addressed at all until 139b.) Just show spikes on your graph at the discreet energies, hf, of possible transitions. Assume that the electron is initially in the ground state and that all transitions from the g.s. to other states are possible.

a) Do a level diagram and show absorption vs frequency for the square well with a barrier (problem 3) for a small barrier, an intermediate barrier and a large barrier case (3 graphs).

b) Do the same for the harmonic oscillator with a barrier (cf. problem 4) (3 cases).

c) Discuss characteristic features and differences in the signatures of the square well and the harmonic oscillator. Discuss the influence of the barrier on the spectroscopic signature.

extra credit

6. a) What is the energy of an electron in an *infinite* square well with L=0.1 nm.
Consider an electron in a *finite* square well, as in problem 1, with $V_0 = 1$ eV and L=0.1 nm.

b) How many bound states does this system have?

c) Determine α and the energy of the ground state for this well. You can do this in closed form by using approximations which involve keeping only terms to linear order in kL for the wavefunction inside the well. That is, $\cos(kL) \approx 1$ and $\sin(kL) \approx kL$. Don't try to keep any more terms or they will screw you up. Match at the boundary and solve. You don't need both boundaries. They are equivalent and the equations from them are redundant.

d) Graph $\psi_1(x)$. How does α^{-1} compare to the well width. What is α^{-1} for L=0.01 nm?

e) How does α depend on L and V_0? *Express E_1 in terms of α^{-1}.* Notice how this is similar in form to $E_1 = \hbar^2\pi^2/(2mL^2)$ (but very different in magnitude).

7. For either the potential of 3) or 4), with a fairly strong barrier, establish the characteristic time for an electron to *"tunnel"* from one side of the barrier to the other. Leave your answer in terms of E_1 and E_2. To do this, begin by showing that you can write a *"mixed state"* that is essentially on just one side of the barrier. Then calculate $< x >$ or just look at $|\psi(x)|^2$ as a function of t and try to find a time when the electron is essentially in a mirror state of the original t=0 one you constructed that is on the opposite side of the barrier. How does this "time to tunnel" acquire its dependence on barrier height? How does $E_2 - E_1$ depend on barrier height?