This is a closed book exam, except that you may consult your single 8½-inch by 11-inch sheet of notes if you have prepared one. You may use a calculator. You’ll have an hour and ten minutes to work the five problems. Each problem is worth 10 points. If you limit yourself to 12 minutes per problem, you will have 10 minutes left over to check your work.

Please do your work on these sheets. You may use sheets from the pile of scratch paper at the front of the room if you need more space.

In each problem be sure to show the logical steps of your reasoning. Getting the reasoning correct is just as important as getting the correct answer. If there is a numerical answer, be sure to include the appropriate units.

\[ \varepsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 / \text{N-m}^2, \text{ or} \]
\[ k = 1/(4\pi\varepsilon_0) \approx 9 \times 10^9 \text{ N-m}^2/\text{C}^2 \]
\[ e = 1.6 \times 10^{-19} \text{ Coulombs} \]
1 Ångstrom (= 1 Å ) = 10^{-10} meters; 1 \( \mu \)F = 10^{-6} Farads.

**Problem 1.** Suppose there is an electron at the origin.

(4) (a) What is the magnitude of the electric field \( E \) at the point \( x = A = 2 \text{ Ångstroms} \)? Please also draw an arrow on the diagram showing the direction of \( E \).

\[
E = k \frac{Q}{r^2} = 9 \times 10^9 \frac{1.6 \times 10^{-19}}{(2 \times 10^{-10})^2} = 3.6 \times 10^{10} \text{ volts/meter (or N/C)}
\]

(3) (b) What is the value of the electric potential \( V \) at the point \( x = A \)? You may assume that \( V = 0 \) infinitely far from the electron.

\[
V = k \frac{Q}{r} = -k \frac{e}{x} = -9 \times 10^9 \frac{1.6 \times 10^{-19}}{2 \times 10^{-10}} = -7.2 \text{ volts}
\]

(3) (c) Now suppose a second electron is placed at \( x = A \). What is the potential energy of the resulting configuration of the two electrons? Make the same assumption as in (b), *i.e.*, \[ \text{potential energy} = k \frac{Q_1 Q_2}{r} \]
that \( V = 0 \) infinitely far from the electrons. Express your answer in both electron-volts and in Joules. Try to get the right sign for your answer.

Note that it takes work to bring another electron near, so the system will have a positive potential energy; it is like compressing a spring. We have

\[
PE = QV = -eV = +7.2 \text{ electron-volts} = 1.15 \times 10^{-18} \text{ Joules}
\]

**Problem 2.** Now suppose a proton is placed at
\[
y = B = 2 \, \text{Å}, \text{ and an electron is placed at } x = A = 2 \, \text{Å}. \text{ (No charge exists at } x = y = 0). 
\]
(2) (a) Show by drawing an arrow on the diagram the direction of the electric field \( \mathbf{E} \) at the origin.

(2) (b) What is the magnitude of \( \mathbf{E} \)?

The diagonal of a square is \( \sqrt{2} \) times a side (by Pythagoras), so using \( E \) from Problem 1, \( E_{\text{total}} = \sqrt{2}E_{\text{electron}} = \sqrt{2}(3.6 \times 10^{10}) \approx 5.1 \times 10^{10} \text{ volts/meter} \).

(3) (c) What is the value of the electric potential \( V \) at the origin? Again assume that \( V = 0 \) infinitely far from the origin.

\( V = 0 \) at the origin because this point is equally distant from the proton and electron, which have equal and opposite charges.

(3) (d) Now imagine a Gaussian surface consisting of a sphere of radius \( R = 3 \, \text{Å} \) centered on the origin. What is the magnitude of the total electric flux \( \Phi \) through this surface?

A Gaussian surface of radius 3 Å will completely surround the two particles, and since Gauss’ Law says the total flux \( \Phi \) through any closed surface is equal to \( 1/\varepsilon_0 \) times the charge inside, \( \Phi = 0 \) because the total charge inside is zero.

**Problem 3.** I have a 10-watt headlamp for my bicycle that is driven by a 7-volt battery. A diagram appears at the right, with the lamp represented by the resistor \( R \).

(3) (a) If the lamp runs for 1 hour, how many Joules of energy are dissipated in the lamp? The lamp dissipates 10 watts of power, so \( P = 10 \) watts, but also \( P = \Delta E/\Delta t \), so

\[
\Delta E = P\Delta t = (10)(3600) = 3.6 \times 10^4 \text{ Joules}
\]
(3) (b) How much current flows in the lamp? The voltage across the lamp is $\Delta V = \varepsilon$, so that the power dissipated by the lamp is $P = I\varepsilon$, so $I = P/\varepsilon = 10\text{watts/7volts} = 1.43$ Amperes.

(2) (c) What is $R$, the resistance of the lamp?

$$P = \frac{(\Delta V)^2}{R}, \text{ so } R = \frac{(\Delta V)^2}{P} = \frac{\varepsilon^2}{P} = \frac{49}{10} = 4.9 \text{ ohms}$$

(2) (d) If two lamps, each of resistance $R$, are connected in series to the battery, how much power does each lamp dissipate?

Here each resistor represents one lamp just like the one described at the start of this problem. For the total power dissipated in the circuit at the left, we have $P_{\text{total}} = \varepsilon^2/2R$, which is half the power dissipated when there is only one lamp in the circuit. Now half of this total power is dissipated in each lamp; therefore the power dissipated in each lamp is only a quarter of the 10 watts, or 2.5 watts.

Problem 4. Suppose a capacitor is constructed from two circular metal plates, as shown by the diagram at the right—much like the one we used in our lecture demonstration. The plate separation $d = 1 \text{ cm}$, and the radius of each plate is $r = 10 \text{ cm}$. Closing the switch will apply a voltage of 100 volts to the capacitor plates.

(3) (a) What is the magnitude of the electric field $E$ in the region between the plates when the switch is closed? In which direction does $E$ point?

$$E = \frac{\Delta V}{d} = \frac{100}{0.01} = 10^4 \text{ volts/meter}$$

(2) (b) What is $Q$, the charge on the positive plate?

Since $E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\pi r^2 \varepsilon_0}$, $Q = \pi r^2 \varepsilon_0 E = \pi (0.1)^2 (8.85 \times 10^{-12})(10^4)$, or $Q = 2.78 \times 10^{-9} \text{ C}$
(2) (c) What is \( C \), the capacitance of this capacitor? (Include the units.)

\[
C \equiv \frac{Q}{\Delta V} = \frac{\pi r^2 \varepsilon_0}{d} = \frac{\pi (0.1)^2 (8.85 \times 10^{-12})}{0.01} = 2.78 \times 10^{-11} \text{ Farad}
\]

(3) (d) The switch is now opened. If the plate separation is then halved to 0.5 cm, what happens to (i) the field between the plates, and (ii) the voltage between the plates? Opening the switch will fix \( Q \) at its initial value, so (i) \( E \) will remain the same (see b); and (ii) \( \Delta V \) will therefore be halved (see a).

**Problem 5.** For this problem, \( R = 1000 \Omega \), and \( C = 0.01 \mu F \).

(5) (a) What is the capacitance between A and B for the diagram shown at the right? The two capacitors in the lower branch are in series, so we add their inverses:

\[
\frac{1}{C_{\text{lower}}} = \frac{1}{C} + \frac{1}{C} = \frac{2}{C} \quad \text{so} \quad C_{\text{lower}} = \frac{C}{2};
\]

This is in parallel with \( C_{\text{upper}} = C \), so

\[
C_{AB} = C + \frac{C}{2} = \frac{3C}{2} = 0.015 \mu F
\]

(5) (b) What is the resistance between A and B for the diagram shown at the right? The two resistors in parallel combine like this:

\[
\frac{1}{R_{\text{left}}} = \frac{1}{R} + \frac{1}{R} = \frac{2}{R} \quad \text{so} \quad R_{\text{left}} = \frac{R}{2};
\]

This then adds in series with the resistor on the right, so that

\[
R_{AB} = \frac{R}{2} + R = \frac{3R}{2} = 1500 \Omega
\]