Problem 1. The Hubble telescope was carried into Earth’s orbit and released by the space shuttle Discovery. Its primary concave mirror has a focal length of approximately 12 meters. For part (a), choose the correct answer, making sure to explain your reasoning.

(5) (a) When the Hubble telescope is focused on a very distant object, the image from its primary mirror is: (i) virtual and erect; (ii) virtual and inverted; (iii) real and erect; (iv) real and inverted.

(iv) is the correct answer: The image of a distant object that is of course much farther than the focal plane from the mirror must be real and inverted. If the object is a distance \(d_1 \gg f\) from the mirror, the image appears at a distance \(d_2\) such that

\[
\frac{1}{d_2} = \frac{1}{f} - \frac{1}{d_1} \approx \frac{1}{f} \quad \text{so} \quad d_2 \approx f \quad \text{and is positive, in front of mirror}
\]
(5) (b) If you were to somehow place yourself a distance of 2 meters in front of the primary mirror, where would the image of yourself be formed, and by what factor would you be magnified (or shrunk)?

Now \( d_1 = 2 \text{ meters} = \frac{f}{6} \), so

\[
\frac{1}{d_2} = \frac{1}{f} - \frac{6}{f} = -\frac{5}{f}, \quad \text{and} \quad d_2 = -\frac{f}{5} = -\frac{12}{5} = -2.4 \text{ meters}.
\]

The image is therefore behind the mirror, is virtual and upright, and magnified by a factor \( d_2/d_1 = 2.4/2.0 = 1.2 \). Appropriate diagrams may also be used to illustrate answers to (a) and (b).

---

**Problem 2.** A ray of light is incident on the 1st surface of a \( 45^\circ \) prism as shown in the diagram above, making an angle of incidence of \( 45^\circ \) with respect to the normal to this surface. The refractive index of the prism glass is \( n = 1.4 \) as shown in the diagram. In what follows, you may assume that the refractive index of the air surrounding the prism is \( n_{\text{air}} = 1.0 \).

(2) (a) A fraction of the incident ray will be reflected at the 1st surface. Sketch on the diagram the ray that is reflected, indicating the angle of reflection.

The ray reflected from the first surface makes a \( 45^\circ \) angle with the normal to that surface, as shown in the diagram above.

(3) (b) A fraction of the incident ray will be refracted at the 1st surface. The angle of refraction at this surface is (i) about \( 45^\circ \), (ii) about \( 30^\circ \), (iii) about \( 82^\circ \). Choose the correct answer, and sketch the refracted ray on the diagram.

(ii) must be the correct answer, since the ray will be bent toward the normal with \( n > 1 \). We must have (using Snell’s Law):

\[
n \sin \theta_2 = \sin \theta_1 = \sin 45^\circ = 1/\sqrt{2}, \quad \text{so} \quad \theta_2 = \sin^{-1}[1/\sqrt{2}] \approx 30.34^\circ
\]

(\( \theta_1 \) and \( \theta_2 \) are the angles of incidence and refraction at the 1st surface.)
(3) (c) The angle of incidence of the ray that strikes the 2nd surface of the prism is (i) about 30°, (ii) about 15°, (iii) about 45°. Choose the correct answer, being sure to justify your reasoning.

(ii) must be the correct answer, as seen from the diagram. Doing the geometry: The normals to the two surfaces make an angle of $180° - 45° = 135°$ with respect to each other, so $\theta_3 = 180° - 135° - \theta_2 = 45° - 30.34° = 14.66° \approx 15°$. ($\theta_3$ is the angle of incidence at the 2nd surface.)

(2) (d) A fraction of the ray may be refracted at the 2nd surface. The angle of refraction at this surface is (i) about 21°, (ii) about 30°, (iii) about 45°, (iv) there is no refracted ray, since the ray is totally internally reflected. Choose the correct answer, and sketch this refracted ray if it exists.

Here we use Snell’s Law again: $n \sin \theta_3 = \sin \theta_4$, where $\theta_4$ is the final angle of refraction at the 2nd surface. Hence $\sin \theta_4 = 1.4 \sin(14.66°) \approx 0.354$, so $\theta_4 = \sin^{-1}(0.354) \approx 21°$, so (i) must be the correct answer. Note that the ray refracted at the 2nd surface must be bent away from the normal.

Problem 3. Consider the circuit shown at the right, consisting of a resistor $R = 1$ megohm, a capacitor $C = 1$ microfarad, a 20-volt battery and a switch. The switch is initially in position 1 as shown. At time $t = 0$, the switch is moved to position 2.

(5) (a) Sketch a graph showing the voltage at point $A$ vs the time $t$, starting from $t < 0$ and extending for some time $t > 0$. This is the voltage across the resistor, since the voltage is zero at the “ground”. Be sure to show the (approximate) scales on both axes.

Before the switch is moved, $V_A = 0$. When the switch is moved to position 2, a current $I = \epsilon/R$ initially flows, charging the capacitor, so $V_A$ jumps to $IR = \epsilon = 20$ volts. As the capacitor charges, the current drops exponentially to zero, with a time constant $RC = 1$ second, and hence $V_A = IR$ does also.
(5) (b) Now suppose the switch is moved back to position 1, after having been at position 2 for a long time. Again sketch a graph showing the voltage at point $A$ as a function of $t$. If you like you may show this on the same axes you used for part (a).

Now if the switch is moved back to position 1, the capacitor will discharge, so the current will initially flow in the opposite direction to the charging direction (counter-clockwise around the circuit), so $V_A$ will jump to $-20$ volts before again tending exponentially to zero, as shown in the second part of the plot on the previous page.

---

Problem 4. A rose is placed 40 cm to the left of a convex lens whose focal length $f = 30$ cm.

(4) (a) Where is the image of the rose formed?

This is a converging lens, with $f = 30$ cm. Let the object distance be $d_1 = 40$ cm, and let’s call the image distance $d_2$.

$$\frac{1}{d_2} = \frac{1}{f} - \frac{1}{d_1} = \frac{1}{30} - \frac{1}{40} = \frac{10}{1200}, \text{ so } d_2 = 120 \text{ cm}$$

that is, 120 cm to the right of the lens. (It’s a real, inverted image.)

(3) (b) If the rose is 10 cm high, what is the height of its image?

The image will be larger than the rose by the factor $d_2/d_1 = 120/40 = 3$, so the image will be $3 \times 10 = 30$ cm high.

(3) (c) If the rose is now moved toward the lens, up to a distance of, say, 10 cm to the left of the lens, tell me, qualitatively, how its image moves.

As the rose is moved to the right toward the lens, its image will move to the right, growing larger and larger as the rose nears the focal plane of the lens. When the rose crosses the focal plane, its image will switch to a large virtual upright image to the left of the lens, moving toward the lens as the rose moves to the right. (When $d_1 = 10$ cm, we can use the lens formula again to find that $d_2 = -13.3$ cm, and magnified by a factor 1.33.)
Problem 5. (True or False) For each of the following statements, tell me whether you think it is true or false, and write a sentence or two, or make a diagram, to justify your answer. There is no credit unless you justify your answer.

The bottom half of a rectangular loop of wire is in a magnetic field \( B \) as shown in the sketch at the right, while the top half remains out of the field. The direction of \( B \) is into the paper. All the questions refer to this apparatus.

(2.5) (a) Pulling the loop upwards out of the field will induce a clockwise current in the loop.

Pulling the loop upwards (towards the top of the page) will decrease the flux of \( B \) through the loop, and Faraday’s Law implies that a current will flow so as to oppose this decrease, trying to keep the flux constant. A clockwise current \( I \) would accomplish this, so the statement is **TRUE**.

Alternatively, we may invoke the Lorentz force law, which says that the magnetic force on a charge in the bottom leg that is moving upward must be to the left, so causing a clockwise current in the loop. (Forces on charges in the vertical legs do not cause a current to flow since such forces are sideways to these legs.)

(2.5) (b) Turning the magnetic field off will induce a counter-clockwise current in the loop.

Turning \( B \) off also decreases the flux of \( B \) through the loop, so this is equivalent to (a) and a clockwise current will flow in the loop, and therefore this statement is **FALSE**.

(2.5) (c) If the loop is moved back and forth in its own plane at an oscillating frequency \( f \), there will be an alternating current induced in the loop at that frequency.

Moving the loop back and forth (i.e., sideways) in its own plane (i.e., in the plane of the paper) does not change the flux of \( B \) through the loop, so no currents are induced in the loop. Hence this statement is **FALSE**.

(2.5) (d) If a counter-clockwise current is made to flow in the loop (by inserting a battery in the loop, say), there will be a upwards force on the loop.

A counter-clockwise current causes charges to move to the right in the bottom leg, so the force on those charges is upwards. Therefore this statement is **TRUE**. (The sideways forces on the left and right legs are equal and opposite and so cancel out.)