Homework Set 10 solutions

1. Page 886, Problem 17. Add: Illustrate the motion of such a particle by sketching a momentum-energy diagram (a plot of $E$ vs $pc$) with an appropriate vector to represent this particle. You may assume the mass of such a particle to be $m$, and so be sure to include in your sketch the “hyperbola of mass-$m$ particles”.

$$p = \frac{mv}{\sqrt{1 - v^2/c^2}} = 3mv \quad \text{so} \quad 1 - v^2/c^2 = 1/9, \quad \text{or} \quad \frac{v}{c} = \frac{\sqrt{8}}{3} = \frac{2\sqrt{2}}{3} = 0.9428$$

Below is a momentum-energy diagram that illustrates such a particle. [Note that $E - mc^2 = K = 2mc^2$, illustrated correctly on this diagram.] The hyperbola of mass-$m$ particles is shown: $E^2 - p^2c^2 = m^2c^4$, and the vector representing our particle has its arrowhead ending on this hyperbola.

2. Consider an electron. (a) What is the rest energy of this electron, both in Joules and keV? (b) If this electron were accelerated by a potential difference $\Delta V = 100$ kilovolts ($10^5$ volts), what would its kinetic energy $K$ be, both in Joules and keV? (c) What would its total energy $E$ be, both in Joules and keV? (d) What would its (relativistic) momentum be, both in kilogram-meters per second, and in keV/c? [Ans: 335 keV/c] (e) What would its velocity be, both in meters per second, and as a fraction of $c$? (f) Sketch a momentum-energy diagram showing the motion of this electron, with both the energy axis
$E$ and the momentum axis $pc$ scaled in keV. The moving electron should be represented by a vector on this diagram, and it should include (as in the previous problem) the appropriate hyperbola. It should also illustrate the magnitudes of $E$, $K$ and $pc$.

(a) The rest energy $mc^2 = (9.109 \times 10^{-31})(8.99 \times 10^{16}) = 8.189 \times 10^{-14}$ Joules, or $5.11 \times 10^5$ eV, or $511$ keV.

(b) If the electron is accelerated by $100$ kV, it will have a kinetic energy $K = 100$ keV = $1.6 \times 10^{-14}$ Joules.

(c) The total energy of the electron is $E = mc^2 + K = 611$ keV, or $9.78 \times 10^{-14}$ Joules.

(d) The momentum of the electron is most easily found from the equation for the invariant hyperbola: $E^2 - p^2c^2 = m^2c^4$, or

$$p^2c^2 = E^2 - m^2c^4 = 611^2 - 511^2 = 112200 \text{ (keV)}^2$$

so $pc = \sqrt{112200} = 335$ keV

and therefore $p = 335 \text{ keV}/c = 1.79 \times 10^{-22}$ kg-m/sec

(e) The speed of the electron is most easily found from noting that

$$\frac{\nu}{c} = \frac{pc}{E} = \frac{335}{611} = 0.548 \text{ so } \nu = 0.548c = 1.64 \times 10^8 \text{ m/sec}$$

(f) Here is a momentum-energy diagram illustrating all of the above:

![Momentum-Energy Diagram](image_url)

The moving electron is represented by the arrow, just as in the previous problem, with the arrow point on the hyperbola of mass-m particles. Note that the electron-volt units are much more convenient than our usual SI units, and lend greater understanding to the situation.
3. Page 944, Problem 12. Be sure to illustrate the process with an appropriate energy-level diagram.

To find the wavelengths $\lambda_{32}$ and $\lambda_{21}$ (see the figure at the left), we first find the energy level differences:

\[ E_3 - E_2 = -E_1 \left( \frac{1}{9} - \frac{1}{4} \right) = 1.889 \text{ eV} = \frac{hc}{\lambda_{32}} \]

(Note $E_1 = 13.6 \text{ eV}$.) Thus, with $1.6 \times 10^{-19} \text{ J/eV}$, $h = 6.626 \times 10^{-34} \text{ J}$, and $c = 3 \times 10^8 \text{ m/sec}$, we find $\lambda_{32} = 6.577 \times 10^{-7} \text{ meters}$, or 6577 Å.

Similarly, $E_2 - E_1 = 10.2 \text{ eV}$, and $\lambda_{21} = 1218 \text{ Å}.$

4. Page 944, Problem 14. Hint: For this energy, what must be the value of $n$, the principal quantum number?

The value of the principal quantum number $n$ may be determined using Eq. 30.10 in the text, with $r = 4.761 \text{ Å}$, to find

\[ n = \left( \frac{4.761 \times 10^{-10}}{5.29 \times 10^{-11}} \right)^{1/2} = 3 \]

Hence

\[ E = -\frac{E_1}{n^2} = -\frac{13.6}{9} = -1.51 \text{ eV} \]