Problem #1: Ranking  

(Note: This problem does not require any calculations, beyond very simple ones that you can do in your head.)

Part I: 
The diagrams below show overhead views of five merry-go-rounds, each of radius $R = 2$ meters and mass $M = 80$ kg. Two passengers stand on each merry-go-round: Ann, of mass 60 kg, and Bill, of mass 100 kg. In each situation, the passengers are standing in different locations on the merry-go-round.

Each merry-go-round has a rocket engine on its outer edge, applying a constant force $F = 200$ Newtons, in the direction shown.

Neglect friction and air resistance.

Rank the five objects according to the magnitude of their angular acceleration $\alpha$, from largest $\alpha$ to smallest. Indicate any ties.
Part II:
Three people -- Lou, of mass 80 kg, Mary, of mass 60 kg, and Nick, of mass 100 kg -- are wearing frictionless roller skates. In frame A below, Lou is moving east at 7 m/s, while Mary and Nick are at rest.

Lou then crashes into Mary. In frame B, Lou and Mary are moving east together. They then crash into Nick. In frame C, all three skaters are moving east as one "object".

Finally, they crash into a tree, which they all grab hold of, and come to rest.

Frame A:

Frame B:

Frame C:

Frame D:

Rank the four frames according to the magnitude of the total momentum of the three people, from greatest total momentum to least. Indicate any ties.

\[ \text{greatest total } p \quad \text{to} \quad \text{smallest total } p \]

Rank the four frames according to the total kinetic energy of the three people, from greatest total kinetic energy to least. Indicate any ties.

\[ \text{greatest total KE} \quad \text{to} \quad \text{smallest total KE} \]
Problem #3: True / False

Mark each statement as "True" or "False". Then, if the statement is false, modify it (by adding, deleting, or changing a few words) so that it becomes true.

1) In a perfectly elastic collision between two objects, the total kinetic energy of the objects after the collision will be greater than their total kinetic energy before the collision.

2) If two objects collide and stick together, then some of their momentum will be transformed into heat.

3) If the total torque on a spinning object is zero, then the object will continue to spin at a constant angular velocity.

4) If you shorten the length of a pendulum, its period will increase.

(continued on next page...)
5) If you increase the amplitude of a mass/spring oscillator, its frequency will increase.

6) If a mosquito flies in a perfect circular path at constant speed, then it is always moving directly toward the center of the circle.

7) Impulse is important because it tells you how much energy a force adds or removes from a system.

8) The total energy of the universe can never change.
Problem #3: Sanity Check

(20 points)

As always with sanity checks, do not try to solve this problem directly (unless you want to try for extra credit at the end.) Instead, read and think about the situation described, and study the five equations proposed as possible answers. At least four of these five equations are flawed in some way, and cannot possibly be the right answer; your job is to explain what's wrong with them.

The problem:
An airplane propeller consists of a uniform rod of mass $M$ and length $L$, free to rotate without friction about an axis through its center, as shown below. The propeller is initially at rest, when it is struck by a small blob of clay of mass $m$, moving horizontally at speed $v$. The clay strikes the propeller at a distance $y$ from the axis of rotation as shown below. The clay sticks to the propeller, and its impact sets the propeller rotating with angular velocity $\omega$.

You may assume that the clay's mass $m$ is much less than the propeller's mass $M$, and may neglect all forms of friction and air resistance.

Your goal is to find an equation for $\omega$.

The possible equations:

1. $\omega = \frac{12 m v y}{M L^2}$

2. $\omega = \frac{m v L}{3 (m + M) y^2}$

3. $\omega = \frac{m v y^2}{(m^2 + M) L^3}$

4. $\omega = \frac{2 m v L y}{m + M}$

5. $\omega = \frac{3 M v y}{m L^2}$
a) If the mass $M$ of the propeller were increased, how should this affect the propeller's final angular velocity $\omega$?

Which equation(s), if any, does this eliminate? Briefly justify your answer.

b) If the blob of clay were to strike the propeller very near the axis of rotation (so that $y \to 0$), how should this affect the propeller's final angular velocity $\omega$?

Which equation(s), if any, does this eliminate? Briefly justify your answer.

c) If the length $L$ of the propeller were increased, while $M$, $y$, and all other variables remained constant, how should this affect the propeller's final angular velocity $\omega$? Briefly explain your reasoning.

Which equation(s), if any, does this eliminate? Briefly justify your answer.

d) What units should $\omega$ have?

Which equation(s), if any, does this eliminate?

e) Which equation(s), if any, might still be the correct answer?

Extra Credit: (use the back of the facing page <--------)

Describe at least one additional sanity check which could be applied to this problem. Explain how the correct equation should behave under this check, and tell which equation(s), if any, it eliminates.
Problem #4: (25 points)

A ball of mass $M = 5$ kg is attached to a string of length $L = 2$ meters. The other end of the string is tied to a fixed pivot point, and the ball whirls in a vertical circle about this pivot point:

When the ball passes the lowest point in the circle (point A in the above diagram), the tension in the string is $F_T = 300$ Newtons.

a) Calculate the ball's speed $v_A$ as it passes point A. (Hint: a free-body diagram might help.)

Just as it passes point A at the bottom of its circular path, the ball is struck by a blob of clay of mass $m = 2$ kg, and the two objects stick together.

The instant before the collision, the blob of clay is moving in the direction shown below, $30^\circ$ to the left of vertical, at an unknown speed $v_{blob}$. The instant after the collision, the blob and the ball are moving together, directly upward. (Naturally, this means that the string goes slack, and plays no further role in the problem.)

b) Calculate the blob's speed $v_{blob}$ the instant before the collision.

c) Calculate the speed $v_f$ of the blob and the ball the instant after the collision.
Problem 10: The Hour That Stretches

Suppose you own a clock that keeps time by means of a simple pendulum of length $L = 25$ cm, with a bob of mass $m = 100$ g.

a) What is the period of this pendulum's oscillations?

Now you take this clock with you when you move to Mars, where the local gravitational field is weaker than that of the Earth. (On Mars, $g = 3.6 \text{ m/s}^2$.)

b) On Mars, what is the period of the pendulum's oscillations? How will this affect the clock's usefulness as a timekeeping tool?

One of your friends offers to try to get the clock working the way it did on Earth, by changing the mass of the pendulum bob.

c) Will his plan work? If not, why not? If so, what new mass will he have to use for the pendulum bob?

Another of your friends offers to try to get the clock working the way it did on Earth, by changing the length of the pendulum.

d) Will her plan work? If not, why not? If so, what new length will she have to use for the pendulum?
Problem #6: (25 points)

A cart of mass $M = 80$ kg is attached to a spring of unknown spring constant $k$. Initially, the cart is at rest at the spring's equilibrium point, while a dog of mass $m = 20$ kg is running to the right at speed $v_{dog} = 10$ m/s.

The dog jumps aboard the cart, causing it to begin oscillating back and forth around equilibrium with frequency $f = 0.25$ cycles per second.

![Diagram of cart and dog](image)

a) Calculate the spring constant $k$.

b) Calculate the cart's speed $v_{cart}$ the instant after the dog jumps on board.

c) Calculate the amplitude $A$ of the cart's oscillations.
A few seconds after the dog jumps on board, there will be a moment when the cart is instantaneously at rest, at the most extreme left point of an oscillation cycle.

Just at that moment, a steel ball of mass \( m = 20 \text{ kg} \), moving to the right with speed \( v_0 = 12 \text{ m/s} \), strikes the cart and bounces off in a perfectly elastic collision. The dog, unfazed, remains on board the cart:

d) Will the frequency of the cart's oscillations after the ball strikes it be greater, less, or the same as before? Briefly justify your answer.

e) Will the amplitude of the cart's oscillations after the ball strikes it be greater, less, or the same as before? Briefly justify your answer.

Extra Credit: (Use the back of the facing page <---------->
If you answered in part (d) that the frequency of oscillations changed, calculate the new frequency.
If you answered in part (e) that the amplitude of the oscillations changed, calculate the new amplitude.