Problem #1

a) \[ \frac{A}{\text{Strongest}} \quad \frac{B}{\text{next}} \quad \frac{C}{\text{next}} \quad \frac{D}{\text{Weakest}} \]

(Think about how much force is necessary to keep the block from passing through the wall.)

b) \[ \frac{B}{\text{Strongest}} \quad \frac{A}{\text{next}} \quad \frac{C}{\text{next}} \quad \frac{D}{\text{Weakest}} \]

(Think about how much force is necessary to keep the block from sliding.)

c) In diagram D, there is no normal force, therefore no static friction, so the only forces on the book are gravity and the librarian's applied force:

\[ \begin{align*}
\vec{F}_g &= mg \\
\vec{F}_a &= \text{applied force}
\end{align*} \]

\[ \sum F_y = ma_y \]

\[ \Rightarrow \quad \vec{F}_a - mg = 0 \]

\[ m = \frac{F_a}{9} \approx \frac{50N}{10m/s^2} \approx 5 \text{ kg.} \]

Problem #1 (cont.)

a) \[ \frac{A}{\text{highest}} \quad \frac{B}{\text{next}} \quad \frac{C}{\text{next}} \quad \frac{D}{\text{lowest}} \]

\[ (w = \sqrt{\frac{k}{m}}) \]

\[ (v_0 = Aw \Rightarrow A = \frac{v_0}{\sqrt{\frac{m}{k}}}) \]
Problem #2: True / False

Two simple pendulums are set up as shown below (the diagram is not necessarily to scale.) You can assume that as they oscillate, the angle they make from equilibrium is always much smaller than one radian (so that the small-angle approximation always applies.)

Mark each statement as True or False. If the statement is false, then modify it (by adding, removing, or changing a few words) so that it becomes true.

1) If \( m_1 \) is nine times greater than \( m_2 \), then the frequency of pendulum \#1 is one third the frequency of pendulum \#2.
   \[ \text{True} \]

2) If \( L_1 \) is nine times longer than \( L_2 \), then the frequency of pendulum \#1 is one third the frequency of pendulum \#2.
   \[ \text{True} \]

3) If \( m_1 \) is nine times greater than \( m_2 \), then the amplitude of pendulum \#1 is nine times the amplitude of pendulum \#2.
   \[ \text{False} \]

4) Suppose pendulum \#1 is at rest when a moving blob of clay strikes the pendulum bob and sticks to it. Since the collision is inelastic, some of the clay blob's initial momentum is converted into heat.
   \[ \text{False} \]

5) Suppose the two pendulums are identical, except that pendulum \#1 is oscillating with twice the amplitude of pendulum \#2. Both pendulums will then have the same frequency.
   \[ \text{True} \]

6) Suppose the two pendulums are identical, except that pendulum \#1 is on Earth, while pendulum \#2 is on a planet whose gravity is nine times stronger than Earth's. Then the period of pendulum \#1 will be \( \frac{1}{3} \) times longer than the period of pendulum \#2.
   \[ \text{False} \]
a) Units should be meters. This eliminates (d): \[ h = \frac{M}{(m+M)^2} \]

\[
\begin{align*}
\text{m kg} &= \frac{kg}{kg^2} \cdot m \\
\text{m} &= \frac{m}{kg} \quad \text{(not true!)}
\end{align*}
\]

b) \[ \text{if } m \rightarrow 0, \text{ the thief should swing back up to her original altitude (because the safe doesn't affect her motion at all if it has zero mass).} \quad \text{as } m \rightarrow 0, \text{ } h \text{ should } \rightarrow H. \]

This eliminates (a), which implies that as \( m \rightarrow 0, \text{ } h \rightarrow 0 \) (because of the \( m \) in the numerator.)

c) The amount of time that the thief spends studying the situation will have no effect on her final altitude; (c) is eliminated because it mentions \( t \) at all.

d) Eqn (b) is correct.
Problem #4

The situation here is one we've studied before, and seen worked out in Knight's electric collisions section: a one-dimensional, perfectly elastic collision between a moving object and a target which is initially at rest:

\[
\begin{array}{c}
\text{before} \\
0 \rightarrow \boxed{M} \rightarrow v_0 \\
\text{after} \\
0 \rightarrow \boxed{M} \rightarrow v_2
\end{array}
\]

The results we've worked out in the past are:

1. \( V_1 = \left( \frac{m-M}{m+M} \right) v_0 \)
2. \( V_2 = \left( \frac{2m}{m+M} \right) v_0 \)

In this case, we know \( v_0 = 20 \text{ m/s} \), \( v_1 = -10 \text{ m/s} \), \( m = 2 \text{ kg} \), but do not know \( v_2 \) or \( M \).

a) Solve eq. 1 above for \( M \):

\[
(m+M)v_1 = (m-M)v_0
\]

\[
Mu_1 + Mv_0 = mv_0 - mv_1
\]

\[
M = \left( \frac{v_0 - v_1}{v_0 + v_1} \right) m = \left( \frac{20 \text{ m/s} - (-10 \text{ m/s})}{20 \text{ m/s} + (-10 \text{ m/s})} \right) (2 \text{ kg})
\]

\[
M = 6 \text{ kg}
\]

b) Now use eq 2 to get \( v_2 \):

\[
v_2 = \frac{2m}{m+M} v_0 = \left( \frac{2 \cdot 2 \text{ kg}}{2 \text{ kg} + 6 \text{ kg}} \right) (20 \text{ m/s})
\]

\[
v_2 = +10 \text{ m/s}
\]
c) \( f = 3 \text{ cycles/sec} \)
\[ w = 2\pi f = 6\pi \text{ s}^{-1} \]
for a mass-spring system, \( \omega = \sqrt{\frac{k}{m}} \) we'll use \( M \) here, since it is the oscillating mass

\[ k = M \omega^2 \]
\[ k = (6\text{ kg})(6\pi \text{ s}^{-1})^2 = 216\pi^2 \text{ kg/s}^2 \]
\[ k = 2132.1 \text{ kg/s}^2 \]

d) \( V_{\text{max}} = \omega A \) (and the block's post-collision velocity, \( V_0 = \omega A + 10 \text{ m/s} \), is its max velocity for a cycle.)

\[ A = \frac{V_{\text{max}}}{\omega} = \frac{10 \text{ m/s}}{(6\pi \text{ s}^{-1})} \]
\[ A = 0.53 \text{ m} \]
Problem 4.5: The helicopter towing the sled

(i) \[ \overrightarrow{N} \]
\[ m = 100 \text{ kg} \]
\[ T = 1200 \text{ N} \]
\[ \mu_k = 0.2 \]
\[ \theta = 30^\circ \]

(ii) Vector table:

<table>
<thead>
<tr>
<th>Vector</th>
<th>( x )-comp.</th>
<th>( y )-comp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T )</td>
<td>( +T\cos\theta )</td>
<td>( +T\sin\theta )</td>
</tr>
<tr>
<td>( F_k )</td>
<td>( -\mu_k N )</td>
<td>0</td>
</tr>
<tr>
<td>Weight</td>
<td>0</td>
<td>(-mg)</td>
</tr>
</tbody>
</table>

\[ \sum F_y = ma_y \]
\[ N + T\sin\theta - mg = 0 \]
\[ N = mg - T\sin\theta \]
\[ N = (100 \text{ kg})(9.8 \text{ m/s}^2) - (1200 \text{ N})(\frac{1}{2}) \]
\[ N = 380 \text{ newtons} \]

Assuming (for now) that the sled remains on the ice.

(c) \( a_y = 0 \) (assuming the sled remains on the ice).

\( \sum F_x = ma_x \)

\[ T\cos\theta - \mu_k N = ma_x \]
\[ a_x = \frac{T\cos\theta - \mu_k N}{m} \]
\[ a_x = \frac{(1200 \text{ N})(\frac{\sqrt{3}}{2}) - (0.2)(380 \text{ N})}{100 \text{ kg}} \]
\[ a_x = 9.6 \text{ m/s}^2 \]
d) \[ \Delta x = v_x t + \frac{1}{2} a_x t^2 = 0 + \frac{1}{2} (9.6 \text{ m/s}^2) (10 \text{s})^2 \]
\[ \Delta x = 480 \text{ m} \] (and of course, \( \Delta y = 0 \))

e) This gets tricky:

if we use the result from part b,
\[ N = mg - T \sin \theta = (100 \text{ kg}) (9.8 \text{ m/s}^2) - (2400 \text{ N}) (1/2) \]

then we get \( N = -220 \) newtons

in other words, the equation claims the ground is pulling down on the sled (because \( N \) came out negative!)

The ground can't really hold the sled down, of course... so the sled lifts off the ground, and there is no normal force or kinetic friction!

d) So we rework Newton’s laws, with only two forces: tension in the rope, and gravity... and we no longer can assume \( a_y = 0 \).

\[ \sum F_x = ma_x \]
\[ T \cos \theta = ma_x \]
\[ a_x = \frac{T \cos \theta}{m} = \frac{(2400 \text{ N}) (\sqrt{3}/2)}{100 \text{ kg}} \]
\[ a_x = 20.8 \text{ m/s}^2 \]

\[ \Delta x = \frac{1}{2} a_x t^2 = \frac{1}{2} (20.8 \text{ m/s}^2) (10 \text{s})^2 \]
\[ \Delta x = 1040 \text{ m} \]

\[ \sum F_y = ma_y \]
\[ T \sin \theta - mg = ma_y \]
\[ a_y = \frac{T \sin \theta - mg}{m} \]
\[ a_y = 2.2 \text{ m/s}^2 \]

\[ \Delta y = \frac{1}{2} a_y t^2 \]
\[ \Delta y = 110 \text{ m} \]

dist from start = \( \sqrt{(\Delta x)^2 + (\Delta y)^2} = 1046 \text{ m} \)
Firecracker Problem:

An explosion is an inelastic collision run backwards; we use momentum conservation.

\[ \begin{align*}
\mathbf{M} & : 0.05 \text{ kg} \\
V_0 & : 25 \text{ m/s}
\end{align*} \]

\[ m_1 : 0.02 \text{ kg} \]
\[ m_2 : 0.01 \text{ kg} \]
\[ m_3 : 0.02 \text{ kg} \]

\[ V_{1x} : 5 \text{ m/s} \]
\[ V_{1y} : 20 \text{ m/s} \]

(because \( t = 2 \text{ sec} \) must add up to \( 0.05 \text{ kg} \))

\[ \begin{align*}
\mathbf{a) P_{bx}} & = \mathbf{P_{ax}} + \mathbf{F_{extx}} \\
\mathbf{P_{by}} & = \mathbf{P_{ay}} + \mathbf{F_{exty}} \\
-m_2V_2 + m_3V_{3y} & = 0
\end{align*} \]

\[ V_{3x} = \frac{M V_0 + m_1 V_1}{m_3} = 67.5 \text{ m/s} \]

\[ V_{3y} = \frac{m_2 V_2}{m_3} = +10 \text{ m/s} \]

\[ V_{3y} = \sqrt{V_{3x}^2 + V_{3y}^2} = 68.2 \text{ m/s} \]

\[ \theta = \tan^{-1} \left( \frac{V_{3y}}{V_{3x}} \right) = 8.4^\circ \]
b) before explosion: \[ KE = \frac{1}{2} M v_0^2 = 15.6 \text{ J} \]

after explosion: \[ KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \frac{1}{2} m_3 v_3^2 = 48.8 \text{ J} \]

c) The extra energy came from the chemical energy released in the explosion.

d) Easiest fragment is \( \frac{m_1}{\sqrt{v_{01}}} \):
\[
\Delta y = v_{0y} t - \frac{1}{2} g t^2
\]
\[-h = 0 - \frac{1}{2} g t^2
\]
\[h = 45 \text{ m}
\]
\[\Delta y = 0 - 0.1 g t^2
\]
\[6 = \sqrt{\frac{2h}{g}} = 3.0 \text{ s}
\]
Problem 7

a) Projectile motion:

\[ V_{ox} = V_o = ? \]
\[ V_{oy} = 0 \]
\[ \Delta x = s \]
\[ \Delta y = -h \]
\[ a_x = 0 \]
\[ a_y = -g \]

\[ \Delta y = v_{oy} t + \frac{1}{2} a_y t^2 \]
\[ -h = 0 - \frac{1}{2} gt^2 \]
\[ t = \sqrt{\frac{2h}{g}} \]

\[ \Delta x = V_{ox} t + \frac{1}{2} a_x t^2 \]
\[ s = V_o \sqrt{\frac{2h}{g}} + 0 \]

\[ V_\theta = \frac{s \sqrt{g}}{\sqrt{2h}} = \left( \frac{0.8 m}{\sqrt{2 \cdot 1.2 m}} \right) \frac{98 \text{ m/s}^2}{\sqrt{2 \cdot 1.2 \text{ m}}} \]

\[ V_\theta = 1.62 \text{ m/s} \]

b) Energy Cons:

\[ \frac{N}{mg} = \frac{F_k}{\mu_k mg} \]

\[ E_\theta = E_t + W_{nc} \]
\[ \frac{1}{2} m v_\theta^2 = \frac{1}{2} m v_A^2 - \mu_k mg L \]

\[ v_A = \sqrt{v_\theta^2 + 2 \mu_k g L} = 2.23 \text{ m/s} \]

(Not at all.)
**Spacewalk**

\[ a) \quad V = \frac{2\pi b}{T} = \frac{2\pi (30 \text{ m})}{(20 \text{ s})} = 9.42 \text{ m/s} \]

\[ b) \quad F = \frac{mv^2}{b} = ma = \sqrt{ \frac{mv^2}{b} } = 2.86 \text{ N} \]

**Planet fall**

\[ a) \quad \text{Period of Oscillation: } T = \frac{20 \text{ s}}{5 \text{ cycles}} = 4 \text{ s/cycle} \]

\[ \omega = \frac{2\pi}{1} = 1.57 \text{ s}^{-1} \]

For a pendulum, \( \omega = \sqrt{\frac{g}{L}} \) \[ g = L \omega^2 = (1.23 \text{ m/s}^2) \]

\[ b) \]

\[ V_0 = 30 \text{ m/s} \]

\[ V_{0x} = V_0 \cos \theta = 15 \text{ m/s} \]

\[ V_{0y} = V_0 \sin \theta = 26 \text{ m/s} \]

\[ V_y = 0 \quad \text{at peak of flight} \]

\[ a_x = 0 \]

\[ a_y = -g \]

\[ \Delta y = \frac{V_{0y}^2}{2g} = \frac{(15 \text{ m/s})^2}{2(1.23 \text{ m/s}^2)} = 91.5 \text{ m} \]

\[ \theta = 60^\circ \]

\[ c) \quad V_0 = V_{0x} = 15 \text{ m/s} \quad \text{directly to right.} \]
Shaken, Not Squished:

Energy Conservation:

\[ E_A = Mgh \]

\[ E_0 = \frac{1}{2} m v_0^2 + \frac{1}{2} M v_0^2 + mgh \]

\[ E_0 = E_A + \text{Work} \]

assuming no friction or air resistance.

\[ \frac{1}{2} m v_0^2 + \frac{1}{2} M v_0^2 + mgh = Mgh \]

\[ v_0 = \sqrt{\frac{2 (M-m) gh}{M+m}} = \sqrt{\frac{2 (80\text{ kg} - 75\text{ kg}) (9.8\text{ m/s}^2) (50\text{ m})}{(80\text{ kg} + 75\text{ kg})}} \]

\[ v_0 = 5.6\text{ m/s} \]

b) If instead he is moving at half the expected speed (and thus, so is the counterweight):

\[ \text{expected } KE = \frac{1}{2} m v^2 + \frac{1}{2} M v^2 = \frac{1}{2} (75\text{ kg} + 80\text{ kg}) (5.6\text{ m/s})^2 \]

\[ = 24500\text{ J} \]

\[ \text{actual } KE = \frac{1}{2} (75\text{ kg} + 80\text{ kg}) (2.8\text{ m/s})^2 = 612\text{ J} \]

i.e. we're missing 1838 J of energy, presumably the friction in the pulley did -1838 J of work, turning kinetic energy into heat.

C)
Problem #8:

a) Hooke's Law: \[ F = k \cdot L \]
\[ k = \frac{F}{L} = \frac{160 \text{ N}}{0.1 \text{ m}} \]
\[ k = 1600 \text{ N/m} \]

b) \[ w = \sqrt{\frac{k}{m}} = \sqrt{\frac{1600 \text{ N/m}}{4 \text{ kg}}} \]
\[ w = 20 \text{ s}^{-1} \] (if you prefer to give \( f \), then \( f = \frac{w}{2\pi} \))
\[ f = \frac{10}{\pi} \text{ s}^{-1} \]

c) \( A = 0.1 \text{ m} \); that's where he released the mass from rest.

d) \[ v_{\text{max}} = wa = (20 \text{ s}^{-1}) (0.1 \text{ m}) \]
\[ v_{\text{max}} = 2 \text{ m/s} \]

(alternatively, use energy conservation.)