Problem #1

Shown below are five strings of identical mass density, each of which is tied to a wall at one end, while the other end passes over a frictionless pulley and supports a stack of weights. Rank the strings according to the frequency of their fundamental-mode standing waves, from highest frequency to lowest.

\[ V = \frac{\sqrt{F_T}}{\mu} \]

\[ F_T = \text{weight of hanging masses} \]

\[ \lambda_1 = 2 \times \text{length of string} \]

\[ f_i = \frac{V}{\lambda_1} \sim \frac{\sqrt{F_T}}{L} \]

The strings are ranked as follows:

- **B** (highest frequency)
- **A**
- **E**
- **D**
- **C** (lowest frequency)
Problem #1:

Part 1:
Traveling waves move along five strings as shown below. All five strings have the same linear mass density $\mu$, but are not necessarily at the same tension.

1. $f = 100 \text{ cycles/sec}$
2. $f = 200 \text{ cycles/sec}$
3. $f = 200 \text{ cycles/sec}$
4. $f = 100 \text{ cycles/sec}$
5. $f = 100 \text{ cycles/sec}$

Rank the five strings according to their tension, from highest to lowest.

2 highest tension
1
3
5
4 lowest tension

(high-tension strings will have higher wave velocities; use $v = \sqrt{\frac{T}{\mu}}$)
Problem #2: True/False
Mark each statement as true or false.

If a statement is false, then correct it, by adding, deleting, or changing a few words, so that it becomes true.

1) If Alice is standing twice as far from a loudspeaker than in Bob, then Alice will measure the sound waves to have twice the intensity (compared to Bob's measurement).
   
   (fourth)
   
   one fourth

2) If Alice is standing twice as far from a loudspeaker than in Bob, then Alice will measure the sound waves to have the same frequency (compared to Bob's measurement).

   the same

3) Radio waves have a lower frequency than most other types of sound waves.

   light

4) If earthquake waves pass from granite (where they travel quickly) into shale (where they travel more slowly), then their direction of travel might change.

   true.

   (Snells Law!)
3) A wave will experience a half-cycle phase shift when it passes from a medium in which it moves quickly into a medium in which it moves slowly.

4) A lens with a long focal length is more powerful than a lens with a short focal length.

5) When sound passes from a medium in which it moves slowly into a medium in which it moves quickly, its frequency will not change.

   - true

   or virtual

6) A lens with a positive focal length can only cast real images.
Problem #2: True/False

Mark each statement as true or false.

If a statement is false, then correct it, by adding, deleting, or changing a few words, so that it becomes true.

1) In an ideal gas, the average speed of a molecule depends on the temperature, but not on the molecular weight.
   
   **false**

2) There are approximately one mole of people living on the Earth right now.
   
   **false**

Questions 3-6 refer to the following diagram, showing a non-viscous, incompressible fluid moving through a glass tube:

3) The fluid at point B is moving three times faster than the fluid at point A.
   
   **false**
4) The pressure at point B is lower than the pressure at point A.

   false

5) The pressure at point C is equal to the pressure at point A.

   false

6) If the fluid had non-negligible viscosity, then the pressure at point A would be lower than the pressure at point C.

   higher

   false
Problem #3: Sanity Check

As always on sanity check problems, do not try to solve this problem directly (unless you want to try for extra credit at the end.) Instead, think about the physical situation, examine the six equations given as possible answers, and try to reason out why at least five of the equations are impossible or physically absurd.

The problem:
A large concrete-walled aquarium has a triangular plexiglass viewing window in it. The top vertex of the triangle is right at water level as shown below; the height of the window is $h$, and the width of its base is $w$. Assume that the pressure of the air at the water's surface and the pressure of the air on the outside of the window are both $P_{atm}$. Water's density is $\rho$.

What is the net outward force exerted on the window?

The possible answers:

1. $F = \frac{\rho ghw}{6}$
2. $F = \frac{P_{atm} hw}{\sqrt{2}}$
3. $F = \frac{\rho gh^4}{w}$
4. $F = \frac{\rho gw^4}{2h}$
5. $F = \frac{\rho gh^2 w}{3}$
6. $F = \frac{\rho gw^2 h}{2}$
a) What units should the answer have? \[ \text{Newtons} = \frac{\text{kg} \cdot \text{m}}{\text{s}^2} \]

Which equation(s), if any, does this eliminate?

\(\#1\) (which is in \(\frac{\text{kg}}{\text{s}^2}\))

b) If the width \(w\) of the triangular window were increased, while its height \(h\) remained the same, how should the force change? (Should it increase, decrease, or stay the same?)

as \(w\) increases, \(F\) should increase

Which equation(s), if any, does this eliminate? Briefly justify your answer.

\(\#3\) because \(w\) is in the denominator;

This means as \(w\uparrow, F\downarrow\).

c) If the height \(h\) of the window were increased, with its width \(w\) remained unchanged, how should the force change?

as \(h\) increases, \(F\) should increase

Which equation(s), if any, does this eliminate? Briefly justify your answer.

\(\#4\) \(h\) because \(h\) is in the denominator, so as \(h\uparrow, F\downarrow\).
d) If you still have two or more equations remaining, see if you can find a logical flaw in at least one of them. Explain why this equation(s) cannot be the correct answer.

Force on the window must depend on the water's density $\rho$ and on the gravitational field $g$; these are the reasons that there's a higher pressure inside the tank than outside. So (2) must be wrong, since it doesn't depend on $\rho$ or on $g$.

To distinguish between 4.5 and 4.6, we have to decide which variable should be squared: $h$ or $w$. The force depends on $h$ for two different reasons: larger $h$ leads to higher average water pressure, and also a larger window area for the water to push on. So two factors of $h$ make sense; 4.5 is better than 4.6.

e) Which equation(s), if any, might be the correct answer to this problem?

4.5

Extra Credit:
Derive the answer to this problem "for real", from first principles. (Try using logic similar to the dam problem from your homework, with a few important details changed.)
Does your answer match the equation(s), if any, that you selected in part (e)?
Problem #4:
A spring of spring constant $k = 5000$ N/m is attached to the bottom of a tank containing an unknown liquid (not water).

A wooden cube, with density $\rho_w = 400$ kg/m$^3$ and length $L = 0.5$ m on each side, is attached to the spring. As a result, the spring stretches a distance $z = 0.4$ meters as shown below.

a) Calculate the weight of the wooden cube.

$$\text{weight} = Mg = \rho_w V g$$

but $V = L \cdot L \cdot L$, so

$$\text{weight} = \rho_w L^3 g = (400 \text{ kg/m}^3) (0.5 \text{ m})^3 (10 \text{ m/s}^2) = 500 \text{ Newtons}$$

b) Calculate the buoyant force which the liquid exerts on the cube. (Remember: the liquid is not water, and you do not yet know its density.)

While it's true that $F_b = \rho_{fl} V g$, we don't yet know $\rho_{fl}$. So instead we deduce the buoyant force from Newton's Laws:

$$\Sigma F_y = ma_y$$

$$F_b - \rho_w L^3 g - k z = 0$$

$$F_b = k z + \rho_w L^3 g = 2500 \text{ N}$$
c) Calculate the density of the liquid, \( \rho_{fl} \).

Now that we've calculated \( F_b \), we can use:

\[
F_b = \rho_{fl} V g = \rho_{fl} L^3 g
\]

\[
\therefore \quad \rho_{fl} = \frac{F_b}{L^3 g} = \frac{2500 \text{ N}}{(0.5 \text{ m})^3 (10 \text{ m/s}^2)} = 2000 \text{ kg/m}^3
\]

So the liquid is twice as dense as water.

d) If the cube were cut loose from the spring and allowed to float on the surface of the liquid, what fraction of the cube's volume would be above the surface?

\[
\Sigma F_y = m a_y
\]

\[
F_b - Mg = 0
\]

\[
\rho_{fl} V_{sub} g - \rho_{obj} V g = 0
\]

\[
\rho_{fl} V_{sub} g = \rho_{obj} V g
\]

\[
\frac{V_{sub}}{V} = \frac{\rho_{obj}}{\rho_{fl}} = \frac{400 \text{ kg/m}^3}{2000 \text{ kg/m}^3} = \frac{1}{5}
\]
Problem #5  
(20 points)

An ambulance is driving east at a speed $v_A = 40 \text{ m/s}$. Ahead of it, a distance $x = 800 \text{ m}$, a motorcycle also moves east at speed $v_C = 30 \text{ m/s}$.

The ambulance's siren emits sound at frequency $f_0 = 500 \text{ Hz}$.

\[ f_{\text{obs}} = f_{\text{source}} \left( \frac{1 - \frac{v_C}{v_{\text{sound}}}}{1 - \frac{v_A}{v_{\text{sound}}}} \right) = 500 \text{ Hz} \left( \frac{1 - \frac{30 \text{ m/s}}{340 \text{ m/s}}}{1 - \frac{40 \text{ m/s}}{340 \text{ m/s}}} \right) \]

\[ f_{\text{obs}} = 517 \text{ Hz}. \]

A short time later, both vehicles are still moving the same velocity as before, but the ambulance is now only 400 m behind the motorcycle.

b) Does the motorcyclist now hear a higher frequency than you calculated in part a, a lower frequency, or the same? Briefly justify your answer.

Same; Doppler shifts do not depend on distance.

The motorcycle pulls to the side of the road and is now at rest. The ambulance is still moving with its original velocity, and is now only 50 m behind the motorcycle.

c) Does the motorcyclist now hear a higher frequency than you calculated in part a, a lower frequency, or the same? Briefly justify your answer.

Higher; Doppler shift does depend on the motion of both the source and the receiver. Now that $v_C = 0$, the cyclist is no longer lowering the observed frequency by his motion.